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Anomalous behavior of the upper critical field due to energy dependent broadening of the Landau levels

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Abstract

We suggest a new approach to the linearized gap equations in the BCS-theory which enable us to determine the upper critical field in the semi-classical approximation for the model electron energy spectrum. It is shown that for electron systems with relatively strong BCS coupling having the Landau level (LL) spectrum with the level width increasing far from the Fermi level, the upper critical field has anomalous upward curvature. We also discuss strong coupling for energy independent LL broadening. In this case form of the $H_{c2}(T)$ curve is insignificantly different from that obtained in the Werthamer-Helfand-Hohenberg theory.

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The quasi-classical approach developed by Gor'kov [1] and by Werthamer-Helfand-Hohenberg [2] predicts the universal behavior of the upper critical field in extremely type II superconductors with s-wave electron pairing. This theoretical $H_{c2}(T)$-curve which has a negative curvature in all temperature range well describes the experimental data in conventional superconductors. The recent reports on an anomalous dependence of $H_{c2}(T)$ in a number of novel materials [3 - 7] have renewed interest in this problem. The observed upward curvature reported in these publications is found in a startling contrast to the convex quasi-classical form. A few attempts were undertaken to explain this phenomenon. The upward curvature of $H_{c2}(T)$ just below T_c (critical temperature at zero field) have been obtained in the BCS theory with d-wave superconductivity. Some other approaches such as the bipolaron theory of cuprates [8] suggest new non-BSC mechanism of electron pairing. At the same time, the attempts, based on the theory of conventional superconductors, to describe the anomalous behavior by accounting for the Landau level (LL) quantization [10] with relatively strong BCS coupling [11] show a clear upward dependence in $H_{c2}(T)$ only at the very low temperature, $T/T_c \lesssim .1$ giving a support to other theories.

In the present paper we examine in the framework of the BCS approach the effect of the structure of quasi particle spectrum on the temperature dependence of the upper critical field. The developed new representation of the leading term in the Gorkov expansion provides us a simple tool for analysis of different spectrum realizations. In particular, we consider the effect in $H_{c2}(T)$ due to energy dependent broadening of LLs. Such inhomogeneous broadening could arise for example due to stronger influence of crystal lattice potential the electrons with small energy leading, therefore, to deviations from Fermi-liquid theory for excitations well above Fermi level. Among other reasons which could give upward $H_{c2}(T)$-curvature we discuss such ones as the strong BSC coupling, mini-band structure of the electron energy spectrum, and the Fulde-Ferrell-Larkin-Ovchinnikov pairing due to spin-splitting of LLs. In these models it was found that $H_{c2}(T)$ has a form typical for WHH-theory with well known [12] sharp increase at $T \to 0$ due to resonance pairing. This asymptotic concave curvature can be extended to relatively small temperature region, $T/T_c \lesssim .1$ [11] with decreasing of the BSC coupling constant. Note, however, that the predictions in this region are problematic since the contribution from the resonance pairing is very sensitive to fine level structure. Impurity scattering and diamagnetic pair breaking can radically affect the upper critical field [12, 13].

In the lowest LL approximation near transition the condensation energy is determined by the quadratic term in the Gor'kov expansion

$$F = \left(\frac{1}{V} - A\right)\Delta_0^2$$

where V is the BCS constant. The magnitude of A is given by quantum mechanical expression

$$A=\frac{k_BT}{2\pi a_H^2}\sum_{\nu}\int\frac{dk_z}{2\pi}\sum_{n,n'=0}^{\infty}C_{nn'}g_n\left(k_z,i\omega_\nu\right)g_{n'}\left(k_z,-i\omega_\nu\right) \tag{1}$$

Here $g_n\left(k_z,i\omega_\nu\right)$ is the temperature Green function of electron on LL with index n and momentum k_z along magnetic field $\mathbf{H}=(0,0,H)$,

$$g_n^{-1}\left(k_z,i\omega_\nu\right)=\mu-\hbar\omega_c\left(n+1/2\right)-\varepsilon\left(k_z\right)+i\omega_\nu$$

$\omega_\nu=\pi T\left(2\nu+1\right)$ is the Matsubara frequency, and

$$C_{nn'}=\int_0^\infty e^{-2t}L_n\left(t\right)L_{n'}\left(t\right)dt=\frac{(n+n')!}{n!n'!2^{n+n'+1}}$$

is the overlap integral between n and n' orbitals. For isotropic 3D SC the energy in the z-direction is determined as $\varepsilon\left(k_z\right)=k_z^2/2m$. The expression, Eq.(1), is UV divergent. This infinity can be removed by means of restricting the energy of electrons forming the electron pairs with the Debye frequency, ω_D. Usually, in the literature, two schemes of the UV cutoff implementation are considered. In the first one the sum over Matsubara frequency is performed in the energy interval $|\omega_\nu|\leq\omega_D$ with infinite summation over LLs. In the alternative scheme, the sums over LLs are restricted by $|n-n_F|\leq n_D\equiv\omega_D/\hbar\omega_c$ (in neglecting corrections due to z-direction) Both approaches are equivalent at high temperatures and give qualitatively similar results (with a small number difference) in the low temperature region (see below and [14]).

The parameters of the mean field transition are determined from equation

$$A=\frac{1}{V}\equiv\frac{D_{3d}}{\lambda} \tag{2}$$

where A is a function of temperature and magnetic field and $\lambda=VD_{3d}$. At zero magnetic field the last equation yields maximal critical temperature, $T_c\left(0\right)=T_c$, which is reduced with an increase of magnetic field. Below we discuss shape of the $T_c\left(H\right)$-curve following from solution to Eq.(1) for different models of the electron density of states.

It is convenient to write down Eq.(1) in another equivalent form. Considering, at first, the finite Matsubara sum and using integral representation for Green functions,

$$\left[n_F-n-x^2\pm i\omega\right]^{-1}=\int_0^\infty d\tau e^{\pm i\tau\left[n_F-n-x^2+i\omega\right]}$$

one can perform the summation over LLs with help of well known identity,

$$\sum_{n=0}^{\infty} z^n L_n(t) = (1-z)^{-1} \exp\left(\frac{tz}{z-1}\right).$$

The subsequent integration over t transforms Eq.(1) to

$$A = \frac{1}{2\pi} D_{3d} \frac{2\pi T}{\sqrt{\mu\hbar\omega_c}} \sum_{\nu=0}^{\nu_{\max}} \int_{-\infty}^{\infty} dx \int_{0}^{\infty} \frac{d\tau_1 d\tau_2 e^{-\widetilde{\omega}_\nu(\tau_1+\tau_2)+i\left(n_F-x^2\right)(\tau_1-\tau_2)}}{2-e^{-i\tau_1}-e^{i\tau_2}} \tag{3}$$

where

$$\widetilde{\omega}_\nu = \frac{\omega_\nu}{\hbar\omega_c}, \frac{\hbar^2 k_z^2}{2m_z} = \hbar\omega_c x^2, \text{ and } D_{3d} = \frac{mp_F}{2\pi^2\hbar^3}$$

is the density of states per spin on Fermi surface.

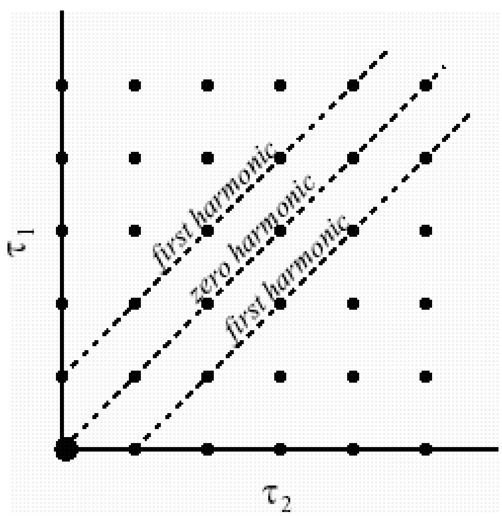

Figure 1: Distributions of poles in Eq. (3). Poles situated on different diagonals contribute to corresponding harmonics of the condensation energy.

The integrand in Eq.(3) has poles in the points of square lattice: $\tau_1 = 2\pi k$, $\tau_2 = 2\pi l$, $k, l = 0, 1, ...$ (Fig. 1). In the quasi-classical region, that is at large values of $n_F \simeq \frac{\hbar\omega_c}{\mu} \gg 1$, the integral Eq.(3) is dominated by the vicinities around the singular points. This important observation enables us to evaluate the contribution of every singularity separately. Another useful property of Eq.(3) is that integration around different diagonals, $\tau_1 = \tau_2 \pm 2\pi m$, gives contributions to different harmonics corresponding to number m. Note that the symmetry between m and $-m$ follows from the invariance of Eq.(3) under complex conjugation and exchange $\tau_1 \leftrightarrows \tau_2$. Therefore, it is enough to estimate the real part of Eq.(3) at $\tau_1 \geq \tau_2$.

For simplicity we restrict our consideration to zero and first harmonics. However one should note that to obtain true shape of the quantum oscillations in H_{c2} at very low temperature, many harmonics should be taken into account.

The zero harmonic is determined by contribution from the lattice points with $k = l$. It is convenient to separate the origin from other diagonal points since these contributions have different physical meanings. The integral from $\tau_1, \tau_2 \ll 1$ is dominated by a great number of the off-diagonal terms and corresponds to the semi-classical contribution to the condensation energy. The integrals around $\tau_1 \sim \tau_2 \sim 2\pi k$ ($k \geqslant 1$) describe the resonance diagonal pairing. These terms are much smaller because of relatively small number of diagonal terms. However, they are singular in the zero temperature limit so that their total value can exceed the off-diagonal terms at $T \rightarrow 0$.

Taking into account a specific feature of the zero point, $k = l = 0$, appearing from the reduced integration region with respect to other singularities (see Fig. 1), the integration around this point can be readily done if we expand the exponential factors in the denominator of Eq.(3) at $\tau_1, \tau_2 \leq \tau_0$ and assume that $\sqrt{n_F}\tau_0 \gg 1$. The result is given by expression

$$\begin{aligned} A_0^{(sc)} &= 2D_{3d}\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}\sum_{\nu=0}^{\nu_{\max}}\Phi\left(\frac{2\omega_\nu}{\sqrt{\mu\hbar\omega_c}}\right) \\ &= D_{3d}\int_0^\infty \frac{dy}{y^2}\frac{\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}y\right)}{\sinh\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}y\right)}\left(1 - e^{-\frac{2\omega_D}{\sqrt{\mu\hbar\omega_c}}y}\right)e^{-y^2}Erfi\left(y\right) \end{aligned} \quad (4)$$

where

$$\Phi\left(\varpi\right) = \int_0^\infty \frac{dy}{y}e^{-\varpi y - y^2}Erfi\left(y\right) \text{ and } Erfi\left(y\right) = \int_0^y \exp\left(x^2\right)dx.$$

Here we have neglected the effects arising from crossing of the sharp cut off by a level from the discrete Matsubara spectrum and have assumed that $T \ll \omega_D$. Eq.(4) was derived by WHH using the quasi-classical representation for Green function.

The similar consideration can be carried out for a 2D system. The result is obtained from Eq.(4) by the replacement, $D_{3d}Erfi(y)/y \to D_{2d} = \frac{m}{2\pi\hbar^2}$, which extracts the contribution from extreme orbits with $k_z = 0$. Thus in 2D system the asymptotic behavior at $y \to \infty$ is determined by the factor $\exp\left(-y^2\right)$ whereas in 3D it is like $\sim 1/y$. It can be shown that quasi 2D systems are characterized by 3D-like damping at relatively small y crossing to an exponential decrease at $y \to \infty$. Note that above damping factor due to energy quantization is important only if the temperature damping is weak at $y \sim 1$. This condition is satisfied only at $2\pi T \lesssim \sqrt{\mu\hbar\omega_c}$. At high temperature there is no difference between 2D and 3D systems.

The finite LLs sum, $\sum_{n=0}^{n_0} = \sum_{n=0}^{\infty} \theta(n_0 - n)$, can be implemented in framework of the suggested approach with help of the integral representation for the θ-function:

$$\theta(x) = \frac{1}{2\pi i}\int \frac{\exp(ix\xi)}{\xi - i\varepsilon} d\xi.$$

After redefinition of z by a factor $e^{-i\xi}$ the sums over LLs are calculated as previously described and the remained integrals over ξ and ξ' are estimated by integration in complex plane. In the quasi classical limit, $n_F \gg 1$, the summation in the energy interval $\omega_1 \leq \hbar\omega_c n \leq \omega_2$ results in

$$\begin{aligned} A_0^{(sc)} &= D_{3d}\int_0^\infty \frac{dy}{y^2}\frac{\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}y\right)}{\sinh\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}y\right)} \times \\ & \frac{1}{\pi}\left[Si\left(\frac{2(\omega_2 - \mu)}{\sqrt{\mu\hbar\omega_c}}y\right) + Si\left(\frac{2(\mu - \omega_1)}{\sqrt{\mu\hbar\omega_c}}y\right)\right] e^{-y^2} Erfi(y) \quad (5) \end{aligned}$$

where

$$Si(x) = \int_0^x \frac{\sin t}{t} dt.$$

Substituting in the above formula $\omega_{1,2} = \mu \mp \omega_D$ and comparing with Eq. (4) one can see that the difference between two schemes of the UV cutoff, consisting in replacement of $\left(1 - \exp\left(-2\omega_D y/\sqrt{\mu\hbar\omega_c}\right)\right)$ by $\frac{2}{\pi} Si\left(2\omega_D y/\sqrt{\mu\hbar\omega_c}\right)$, takes place only for $y \lesssim \sqrt{\mu\hbar\omega_c}/2\omega_D$ and can not qualitatively influence the condensation energy.

To lend a credence to Eq. (3) we have derived low and high temperature asymptotics for the upper critical field paying special attention to the case of small ω_D (strong BCS coupling) as another possible reason for anomalous behavior of the upper critical field. Following to WHH and Gunter-Gruenberg approaches the Matsubara sum restriction have been used.

In the high temperature limit given by condition $\min(\varpi) = \frac{2\pi T}{\sqrt{\mu\hbar\omega_c}} \gtrsim 1$ with $\pi T \ll \omega_D$, the function $\Phi(\varpi)$ is estimated as $\Phi(\varpi) \simeq \frac{1}{2\varpi} - \frac{1}{6\varpi^3}$ by equation

$$\begin{aligned} A \to A_0^{(sc)} &= 2D_{3d} \sum_{\nu=0}^{\nu_{\max}} \left(\frac{1}{2\nu+1} - \frac{1}{6} \frac{\mu\hbar\omega_c}{(\pi T)^2} \frac{1}{(2\nu+1)^3} + .. \right) \\ &\simeq D_{3d} \left(\ln\left(\frac{2e^C\omega_D}{\pi T} \right) - \frac{7\zeta(3)}{48} \frac{\mu\hbar\omega_c}{(\pi T)^2} \right) \end{aligned} \tag{6}$$

which leads to the semi classical equation for the upper critical field [2]:

$$\ln\frac{T_c}{T} = \frac{7\zeta(3)}{48} \frac{\mu\hbar\omega_c}{(\pi T)^2},$$

where ζ is the Riemann zeta function and $C \simeq .578$ is the Eiler constant. The transition temperature at zero magnetic field, T_c, is given by

$$\ln\left(\frac{2e^C\omega_D}{\pi T_c} \right) = \frac{1}{\lambda}.$$

In the case of strong interaction (large λ) when $\frac{\omega_D}{\pi T_c} \lesssim 1$, the low field asymptotic behavior is insignificantly changed to

$$\frac{T}{T_c} = 1 - \frac{\pi^2}{12} \frac{\mu\hbar\omega_c}{(\pi T)^2}$$

with the critical temperature $T_c = \frac{\pi}{4}\lambda\omega_D$ which can exceed the Debye frequency. It should be noted that independently of ω_D the function $T_c(H)$ is linear one at $H \to 0$.

Let us now consider the low temperature limit, $\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}} \lesssim 1$. Note at first that A tends to a constant value at $T \to 0$ so that the upper critical field at zero temperature is given by the integral equation

$$I_{3d}\left(\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}} \right) \equiv \int_0^\infty \frac{dy}{y^2} \left(1 - e^{-\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}} y} \right) e^{-y^2} Erfi(y) = \frac{1}{\lambda} \tag{7}$$

If $\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}} \gtrsim 1$ we can use the asymptotic formula for above integral at large arguments, $I_{3d}(2a) \simeq 1 + \frac{C}{2} + \ln a$, which leads to expression :

$$\hbar\omega_{c2}^{HW} = \frac{e^2}{4e^C} \frac{(\pi T_c)^2}{\mu}.$$

At this magnitude of magnetic field the parameter in above integral,

$$\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}} = \frac{4}{e^{1-C/2}}\frac{\omega_D}{\pi T_c},$$

is large if Debay frequency is not too small.

For 2D system the upper critical field at zero temperature is determined from equation,

$$I_{2d}\left(\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}}\right) \equiv \int_0^\infty \frac{dy}{y}\left(1 - e^{-\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}}y}\right)e^{-y^2} = \ln\left(\frac{2e^C\omega_D}{\pi T_c}\right) \tag{8}$$

which in the limit, $\frac{2\omega_D}{\sqrt{\mu\hbar\omega_{c2}}} \gtrsim 1$, has a solution,

$$\hbar\omega_{c2}^{HW} = \frac{1}{e^C}\frac{(\pi T_c)^2}{\mu}.$$

It is obtained readily from asymptotic expansion, $I_{2d}(2a) \simeq \ln(2a) + \frac{C}{2}$. This expression differs from the above equation for H_{c2} in a 3D system by a number factor of the order $\sim .5$.

For large interaction constant, $\omega_D \sim \pi T_c$, one can approximate LHS of Eq.(7) with a linear function. The obtained critical field

$$\hbar\omega_{c2}(0) = \frac{\pi^3}{4}\frac{\lambda^2\omega_D^2}{\mu} = \frac{\pi^3}{4e^{2+C}}\left(\lambda e^{1/\lambda}\right)^2\hbar\omega_{c2}^{HW} \tag{9}$$

is much larger than it follows from HW theory. The upper critical field grows quadratically with ω_D. Therefore the condition $\hbar\omega_{c2}(0) \ll \omega_D$ restricts the Debye cutoff as $\frac{\omega_D}{\mu} \ll \frac{4}{\pi^3\lambda^2}$. Using the expression for T_c in this limit one can obtain that the ratio

$$\frac{\hbar\omega_{c2}(0)\,\mu}{(\pi T_c)^2} = \frac{4}{\pi}$$

is almost constant for different ω_D. One should stress that to ensure the quasi-classical condition $\hbar\omega_{c2} \ll \mu$, the parameter ω_D/μ should be small, otherwise the system is described by the quantum limit discussed by Rasolt and Tesanovic [15].

The temperature dependent corrections can be estimated using expansion,

$$\frac{1}{x\sinh(x)} = 1 + 2Re\sum_{k=1}\frac{(-1)^k}{k}\int_0^\infty due^{-u\pi k + ixu},$$

with a result

$$\int\limits_0^\infty \frac{dy}{y^2}\left(\frac{\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}y}{\sinh\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}y\right)}-1\right)e^{-y^2}Erfi\left(y\right)\rightarrow-\frac{1}{3}\frac{\left(\pi T\right)^2}{\mu\hbar\omega_c}\ln\left(\frac{\sqrt{\mu\hbar\omega_c}}{e^{C/2-C_1}\pi^2 T}\right) \tag{10}$$

where

$$C_1=\frac{12}{\pi^2}\sum_{k=1}^{\infty}\frac{\left(-1\right)^k}{k^2}\ln k\simeq .123.$$

Thus the dependence $\omega_{c2}\left(T\right)$ in the low temperature region, $\frac{2\pi T}{\sqrt{\mu\hbar\omega_{c2}}}\lesssim 1$, and at large value of ω_D is determined from relation

$$\frac{1}{2}\ln\left(\frac{\omega_{c2}^{HW}}{\omega_c}\right)=\frac{4e^C}{3e^2}\left(\frac{T}{T_c}\right)^2\frac{\omega_{c2}^{HW}}{\omega_c}\ln\left(\frac{e^{1+C_1-C}}{2\pi}\frac{T_c}{T}\left(\frac{\omega_c}{\omega_{c2}^{HW}}\right)^{1/2}\right) \tag{11}$$

Retaining in RHS only the leading term in small parameter $\frac{T}{T_c}$ we can recover the relation obtained by Gorkov [1]: $\ln\left(\frac{H_{c2}(0)}{H_{c2}}\right)=.64\left(\frac{T}{T_c}\right)^2\ln\left(.27\frac{T_c}{T}\right)$. It is interesting to note that the low temperature asymptotic as well as the high temperature limit of the upper critical field do not depend on the interaction constant (for a weak coupling). This universal character of $H_{c2}(T)$-curve holds at $\omega_D\gg\pi T_c$ in all temperature diapason [2]. It is prominent that the curvature of $H_{c2}(T)$ is negative.

In the small ω_D limit we obtain similar dependence,

$$\frac{H_{c2}}{H_{c2}\left(0\right)}=1-\lambda\frac{\pi}{6}\left(\frac{T}{T_c}\right)^2\ln\left(\frac{2e^{C_1}}{\pi^{3/2}e^{C/2}}\frac{T_c}{T}\right), \tag{12}$$

but with a factor proportional to the interaction constant. The numerical solution to Eq. (2) shows that for $\lambda\lesssim 1$ the difference between above two limits smoothly disappears already in a small vicinity above zero temperature. Thus, we conclude that strong energy independent electron-electron coupling can not radically influence the $H_{c2}(T)$-dependence.

At very low temperature, $4\pi^2 T\ll\hbar\omega_c$, the discrete structure of the energy spectrum becomes important and the contribution from other singular points corresponding to resonance pairing strongly increases. Choosing $\tau_1\rightarrow\tau_1+2\pi k$, $\tau_2\rightarrow\tau_2+2\pi k$, where $-\tau_0\leq\tau_1,\tau_2\leq\tau_0$ and $k=1,2...$ numerates the lattice points on main diagonal, we readily calculate the sum over k. The integrals over τ_1,τ_2 are evaluated similar to previous case leading to

$$A_0^{(q)} = D_{3d}\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}}\sum_{\nu=0}^{\nu_{\max}}\frac{e^{-4\pi\widetilde{\omega}_\nu}}{1-e^{-4\pi\widetilde{\omega}_\nu}}\int\limits_{-\sqrt{n_F}\tau_0}^{\sqrt{n_F}\tau_0}\frac{dy}{|y|}e^{-\frac{2\widetilde{\omega}_\nu y}{\sqrt{n_F}}-y^2}Erfi\left(|y|\right)$$

$$\simeq \pi^{3/2}D_{3d}\left(\frac{\hbar\omega_c}{\mu}\right)^{1/2}S_1 \qquad (13)$$

In the integral the parameter $\widetilde{\omega}_\nu$ can be set to zero since the contribution from $\widetilde{\omega}_\nu \sim \sqrt{n_F}$ is strongly suppressed by a small factor $e^{-4\pi\sqrt{n_F}\varpi}$. Estimating the sum,

$$S_1 \equiv \sum_{\nu=0}^{\nu_{\max}}\frac{e^{-4\pi\widetilde{\omega}_\nu}}{1-e^{-4\pi\widetilde{\omega}_\nu}},$$

over Matsubara frequencies we recover the result obtained by GG [12],

$$A_0^{(q)} = \pi^{3/2}D_{3d}\left(\frac{\hbar\omega_c}{\mu}\right)^{1/2}\frac{1}{4\pi}\left(\ln\left(\frac{\hbar\omega_c}{2\pi^2 T}\right)+C\right) \qquad (14)$$

where it is assumed that $\hbar\omega_c \gg 2\pi^2 T$. The resonance pairing correction is small by factor $\sim 1/\sqrt{n_F}$ everywhere except for a limited low temperature region where it is logarithmically diverged.

It is straightforward to estimate the oscillating terms of the SC free energy. The problem is simplified due to factorization of integration and Matsubara summation similar to previous case. Obtained results agree with Ref.([12]) except for the contribution from singular points situated on the axis (Fig. 1) which have not been taken in account in Ref.([12]). Without further comments we present our calculations of the first harmonic contribution to the SC free energy

$$A_1 \simeq \frac{\pi^{1/2}}{4}D_{3d}\left(\frac{\hbar\omega_c}{\mu}\right)\cos\left(2\pi n_F\right)\ln\frac{\hbar\omega_c}{4\pi^2 T} \qquad (15)$$

Note only that it is smaller by factor $1/\sqrt{n_F}$ with respect to zero harmonic.

Similar approach can be applied for more complex electron systems. We have considered the electron density of states with a band energy spectrum along z-direction, $\varepsilon\left(k_z\right) = t_\perp\left(1-\cos\left(k_z d\right)\right)$, where d is the corresponding lattice constant and $t_\perp$ is the mini-band width, and spin-split LLs spectrum with a spatially nonuniform order parameter along the field (Fulde-Ferrel-Larkin-Ovchinnikov approach). In the first case the function $Erfi(y)$ in Eq. (4) should be modified to appropriative one, whereas in the second case the energy spectrum can be treated as a superposition of two independent LL systems. In both cases a convex form of the $H_{c2}(T)$-curves have been obtained. Such a result is rather obvious since in the quasi-classical limit many LLs contribute to pairing making the fine structure of levels less important.

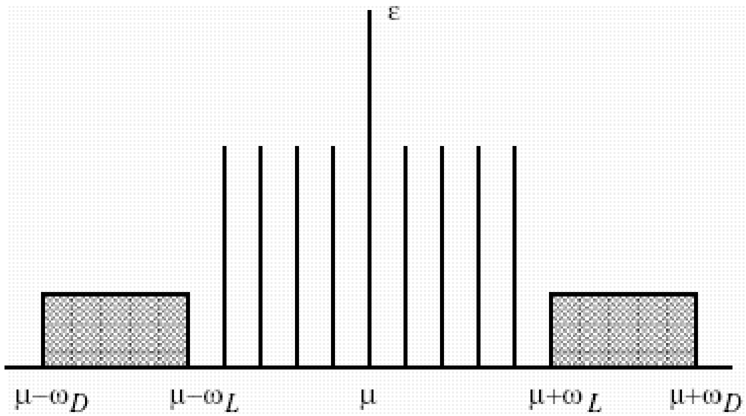

Figure 2: Model density of electron states composed from Landau levels around Fermi surface and continuum at $|\varepsilon - \mu| > \omega_L$.

To simulate an electron system with the level broadening depending on energy we consider a model spectrum shown in Fig. 2. The spectrum is separated in two parts. We assume that in some energy interval around the Fermi surface, $|\varepsilon - \mu| \leq \omega_L < \omega_D$, LLs are well disentangled. For sake of simplicity the width of levels from this part of the spectrum is chosen to be zero. It should be stressed that it is easy to incorporate in the model the levels with a constant width but such a modification do change qualitatively our results. In contrast, LLs which are located beyond ω_L, $\omega_L \leq |\varepsilon - \mu| \leq \omega_D$, are assumed to be strongly overlapped due to some external potential and form a continuum. The contribution to the pairing energy from the level spectrum is described by Eq. (5) with $\omega_{1,2} = \mu \mp \omega_L$. The contribution of continuum can be also described by Eq. (5) where $\omega_2 = \mu + \omega_D$, $\omega_1 = \mu + \omega_L$, and magnetic field tends to zero. Here, we have used that the LL spectrum is transformed to continuum in the zero magnetic field limit. Thus, the upper

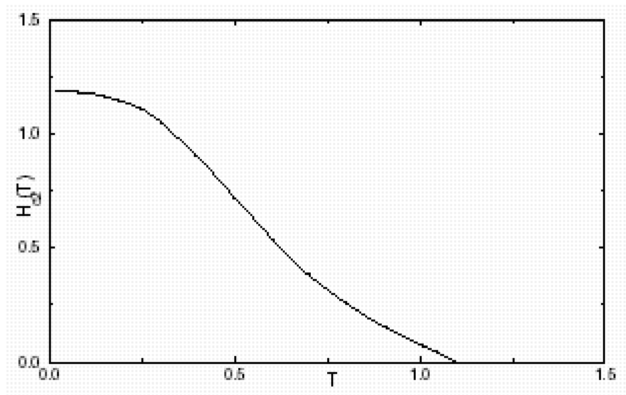

Figure 3: Upper critical field obtained for model density of states with $\lambda = 1$ and $\omega_L/\omega_D = .5$

critical field is determined from equation:

$$\int_0^\infty \frac{dy}{y^2} \frac{\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}} y\right)}{\sinh\left(\frac{2\pi T}{\sqrt{\mu\hbar\omega_c}} y\right)} \frac{2}{\pi} Si\left(\frac{2\omega_L}{\sqrt{\mu\hbar\omega_c}} y\right) e^{-y^2} Erfi\left(y\right) +$$
$$\int_0^\infty \frac{dy}{\sinh\left(y\right)} \frac{2}{\pi}\left[Si\left(\frac{\omega_D}{\pi T} y\right) - Si\left(\frac{\omega_L}{\pi T} y\right)\right] = \frac{1}{\lambda}, \tag{16}$$

where the first term only depends on magnetic field.

It is easy to see that the contribution of the continuum increases with temperature decrease. Such a variation can be considered as a variation of effective interaction constant for the discrete part of the spectrum. This effective interaction also increases with temperature decrease. As a result the upper critical field grows more rapidly than it does for temperature independent interaction constant. Since without continuum $H_{c2}(T)$ is a linear function at $T \to T_c$ it is clear that for a composite spectrum the upper critical field will have a positive curvature just below T_c. At very low temperature the contribution of continuum becomes temperature independent so that $H_{c2}(T)$ has its standard WHH form. An example of the numerical solution to Eq.(16) for $\lambda = 1$ and $\omega_L/\omega_D = .5$ is shown in Fig. 3. The critical field has anomalous behavior up to $.2T_c$.

In summary we note that the anomalous upward dependence of the upper critical field, $H_{c2}(T)$, at high temperature, $T \sim T_c$, can be explained by washing out of LL structure of the electron energy spectrum far from Fermi surface whereas at low temperature the concave form could arise due to the resonance pairing contribution. In contrast, the spin-splitting of LLs and the energy mini-band structure do not affect the classical form of $H_{c2}(T)$ curve.

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