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Streamer propagation in magnetic field

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Abstract

The propagation of a streamer near an insulating surface under the influence of a transverse magnetic field is theoretically investigated. In the weak magnetic field limit it is shown that the trajectory of the streamer has a circular form with a radius much larger than the cyclotron radius of electron. The charge distribution within the streamer head is strongly polarized by the Lorentz force exerted perpendicular to the streamer velocity. A critical magnetic field for the branching of a streamer is estimated. Our results are in a good quantitative agreement with available experimental data.

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Recent experiments [1] on the gas breakdown near an insulating surface in a high magnetic field $\overrightarrow{B}$ have shown new remarkable properties of such discharges. The channel of discharge in a magnetic field appears to have a circular form with the radius R_s several orders of magnitude larger than the electronic cyclotron radius. It decreases with increasing B and reaches $R_s \sim 1$cm at $B \sim 7$T. At higher magnetic field the discharge has a branched structure. These experiments have shown that the streamer propagation cannot be treated as the motion of a charged particle in crossed external electric $\mathcal{E}$ and magnetic B fields.

Interest in the theory of streamers is usually associated with investigation of the gas breakdown phenomena. Streamers were observed also in solids. On an insulating surface near a point electrode, in the region with strong electric field, the discharge has a filamentary structure. The tip of the filament moves with high velocity v_0 ($v_0 \sim 10^8$ cm/s), that exceeds the drift velocity v_d of electrons in the streamer head field $\overrightarrow{E}_s$. The increase of external electric field $\overrightarrow{\mathcal{E}}$ results in penetration of some filaments (so called "leaders") deep into the surrounding gas. Although the plasma parameters in streamers are different from those in leaders, their propagation is associated with the same physical processes and is defined mainly by the parameters of the streamer head. Since we are interested in the behavior of the streamer front only, we will not distinguish here between a streamer and a leader.

The streamer propagation mechanism was suggested by Raether, Loeb, and Meek [2–4], and was further developed by other authors [5–7]. According to this theory, the charged head induces in its vicinity a strong electric field. This field leads to the increase of the electron density beyond the streamer front due to impact ionization. The charge is displaced from this region via Maxwell relaxation. It is assumed that necessary amount of free electrons in front of the streamer front is produced, for example, by the streamer head radiation. A simple model which takes into account only these main processes was considered by M.I. D'yakonov and V.Y. Kachorovskii [8, 9]. They have estimated theoretically the streamer parameters and have shown that the streamer velocity v_0 and radius r_s are changing slowly with external electric field $\overrightarrow{\mathcal{E}}$, e.g. the propagation of the streamer head can be treated as a quasistationary process.

In the present paper we generalize the streamer model [8, 9] to include an external magnetic field. It is assumed that the plasma filaments propagate in the plane perpendicular to the external magnetic field and the streamer parameters do not change along the field direction, Fig. 1. In the weak magnetic field limit, a quasistationary streamer in the coordinate system rotating with a constant angular velocity $\omega_s = v_0/R_s$, proportional to the head charge density, can be considered as a streamer in the absence of magnetic field.

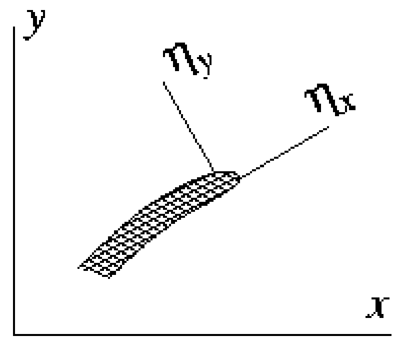

Figure 1: Schematic description of the streamer propagation in the plane perpendicular to the external magnetic field. The coordinate system η rotates with a constant angular velocity $\omega_s = v_0/R_s$, proportional to the head charge density.

Since the energy relaxation time of electrons is much larger than the electron-ion relaxation time, we ignore the gas heating processes. Thus the concentration of atoms changes smoothly at the distance of the order of streamer head size and it is assumed to be constant inside the head. We neglect also the ion drift velocity in comparison with the electron drift velocity $\overrightarrow{v}_d$ and the streamer velocity $\overrightarrow{v}_0$.

The system of equations for the electron density n, the ion density N, and the electric field $\overrightarrow{E}$ is

$$\frac{\partial n}{\partial t} + div\left(n\overrightarrow{v}_d\right) = \beta\left(E\right)n,$$

$$\frac{\partial N}{\partial t} = \beta\left(E\right)n,$$

$$div\overrightarrow{E} = 4\pi\rho\left(x\right),\ rot\overrightarrow{E} = 0, \tag{1}$$

where $\rho\left(x\right) = e\left(N - n\right)$ is the charge distribution, $\beta\left(E\right) = v_d\alpha\left(E\right)$. The impact-ionization coefficient $\alpha\left(E\right)$ increases very sharply with the field and saturates at some field value E_0 [10]

$$\alpha\left(E\right) = \alpha_0 e^{-E_0/E}. \tag{2}$$

We assume for simplicity that the electron drift velocity is proportional to the electric field $\overrightarrow{E}$, $v_{di} = \mu_{ik} E_k$. Without external magnetic field, mobility μ_{ik} is a diagonal tensor, $\mu_{ik} = \mu_0 \delta_{ik}$. In a weak magnetic field, mobility μ_{ik} is a function of $\overrightarrow{B}$, which can be written as

$$\mu_{ik} = \frac{\mu_0}{1+\gamma^2} \left(\delta_{ik} + \gamma \varepsilon_{ik} \right), \tag{3}$$

where ε_{ik} is an antisymmetric tensor in the plane normal to $\overrightarrow{B}$. The parameter $\gamma = \omega_B \tau_{ea}$ ($\gamma \equiv \tan \theta_H$, with θ_H - the Hall angle), where $\omega_B = \frac{eB}{m_e c}$ is the cyclotron frequency and τ_{ea} is the time of electron-atom collisions, is assumed to be small, $\gamma \ll 1$. In what follows we will be interested only in linear corrections in γ to the solution of Eq. (1). Since the parameters α_0, E_0, and v_d depend on γ^2 [11], in our approximation they are magnetic field independent.

Let us consider the streamer propagation Eq. (1) in the coordinate system having the origin at

$$\overrightarrow{r}_s = \frac{\int \overrightarrow{r} \rho \left(\overrightarrow{r}, t \right) d\overrightarrow{r}}{\int \rho \left(\overrightarrow{r}, t \right) d\overrightarrow{r}}$$

and rotating with the angular velocity $\omega(t)$

$$\eta_i = \sum_k \Omega_{ik}(t) \left(x_k - x_{sk}(t) \right). \tag{4}$$

Here $\Omega_{ik}(t)$ is the rotation matrix for the angle $\varphi = \int \omega(t)\, dt$ in the plane perpendicular to $\overrightarrow{B}$:

$$\Omega_{ik}(t) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}. \tag{5}$$

The transformation of derivatives are

$$\frac{\partial}{\partial x_i} \to \sum_k \frac{\partial}{\partial \eta_k} \Omega_{ki}, \qquad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \sum_{ikl} \frac{\partial}{\partial \eta_k} \left[\dot{\Omega}_{ik} \Omega^{-1}_{kl} \eta_l - \Omega_{ki} \dot{x}_{si} \right]. \tag{6}$$

Here dot denotes time derivative. It follows from Eq. (6) that the streamer charge propagates quasistationary if $\sum_k \dot{\Omega}_{ik} \Omega^{-1}_{kl} = \varepsilon_{ik} \omega = const$ and $\sum_i \Omega_{ki} \dot{x}_{si} = v_{0k} = const$. Thus the head of the quasistationary propagating streamer moves with the constant velocity v_0 along a circle with the radius $R_s = v_0/\omega$.

Rewriting Eq. (1) for quasistationary propagation in the rotating system, we obtain

$$\sum_{il} \frac{\partial}{\partial \eta_i} n \left[-v_{0i} + \omega \varepsilon_{il} \eta_l + \mu_0 \left(E_i + \gamma \varepsilon_{il} E_{0l} \right) \right] = \beta \left(E \right) n,$$

$$-\sum_i v_{0i} \frac{\partial N}{\partial \eta_i} = \beta \left(E \right) n,$$

$$\sum_i \frac{\partial E_i}{\partial \eta_i} = 4\pi \rho \left(\eta \right), \quad \sum_{ik} \varepsilon_{ik} \frac{\partial E_i}{\partial \eta_k} = 0. \tag{7}$$

Eq. (7) should be solved with the boundary condition

$$\rho_s v_0 = e n_s \mu_0 E_{s,} \tag{8}$$

which follows from the charge conservation on the surface of the streamer front. Here n_s and $\rho_s = e \left(N - n_s \right)$ are the electron and the charge densities at the front.

The electric field E_{0k}, which appears in the linear in γ term, is the field of a streamer propagation in the absence of external magnetic field. It consists of two parts: the external field $\overrightarrow{\mathcal{E}}$ and the field $\overrightarrow{E}_\rho$ created by the head charge. Field $\overrightarrow{E}_\rho$ is a symmetrical function with respect to the streamer axis. Usually $\left| \overrightarrow{\mathcal{E}} \right|$ is negligible in comparison with $\left| \overrightarrow{E}_\rho \right|$, but near the electrode it can strongly influence the streamer propagation.

We expand $E_{0k} \left(\eta \right)$ near the central point $\eta_i = 0$

$$E_{0k} \left(\eta \right) = E_{0k} \left(0 \right) + a \delta_{kl} \eta_l + b_{kl} \eta_l + D_k, \tag{9}$$

where $a = 2\pi \rho \left(0 \right)$, b_{kl} - is a symmetrical matrix with $Sp \left(b \right) = 0$. The term $E_{0k} \left(0 \right) + b_{kl} \eta$ forms the potential field which satisfies the Laplace equation and can be included into E_k as a correction. The field D_k defined by Eq. (9) is proportional to the deviation of the charge distribution from the uniform one. It is small in the central region and becomes large near the surface of the streamer head. Supposing that the streamer propagation is defined mainly by the central region, one can discard the terms with $\overrightarrow{D}$ while determining the streamer trajectory.

The second term in Eq. (9) leads to the streamer turning. At $\omega = -2\pi \rho \left(0 \right) \gamma \mu_0$ the equations (7) turn to a system of equations describing the quasistationary streamer propagation without magnetic field. Thus streamers moving from cathode and anode will turn in opposite directions with frequency $\omega_s = \left| \omega \right| = \left| 2\pi \rho \left(0 \right) \gamma \mu_0 \right|$.

Introducing Maxwell relaxation time $\tau_m^{-1} = 4\pi\mu_0 e n_s$, one obtains

$$\omega_s \tau_m = \frac{\gamma\rho\,(0)}{2en_s}.$$

If the streamer radius r_s is of the same order of magnitude as the characteristic distance of electric field increase from the internal region to the front, one can estimate r_s as $r_s \simeq \tau_m v_0$ and we have

$$\frac{r_s}{R_s} = \frac{\gamma\rho\,(0)}{2en_s}. \tag{10}$$

This value is very small, since $\gamma \ll 1$ and $\frac{\rho(0)}{en_s} \sim \frac{\mu E_s}{v_0} \ll 1$. The last inequality can be obtained from Eq. (8) at $\rho\,(0) \simeq \rho_s$. Parameter γ is proportional to the magnetic field B, so the streamer radius decreases as $1/B$. This dependence $R_s\,(B)$ differs somewhat from that experimentally observed in [1] $R_s\,(B) \sim 1/B^{\alpha}$, where $\alpha \sim 1.3 - 1.5$. Such disagreement is probably connected with an approximate description of the ionization coefficient and mobility by formulas (2), (3). It must be especially noticeable at high magnetic field.

To evaluate the streamer head charge, we consider the one-dimensional streamer equations (1). Such approximation holds if the width δ of the streamer front is much smaller than the head size r_s: $\delta \ll r_s$. This is the case if the electron density in front of the streamer is much smaller than inside it [9]. Equations (1) at $B = 0$ have a simple analytical solution. Supposing that the streamer moves along x-axis and choosing the boundary conditions as $n\,(-\infty) = n_\infty$, $E\,(-\infty) = 0$, and $n\,(E = E_s) = 0$, we can easily obtain the relation between the equilibrium electron density n_∞ and the electric field E_s at the front

$$\frac{n_\infty}{n_0} = \frac{1}{1 - \frac{\mu_0 E_s}{v_0}} \int\limits_0^{E_s/E_0} e^{-1/x} dx. \tag{11}$$

Here $n_0 = \alpha_0/4\pi e$. This solution describes a plane wave with a narrow front if $\mu_0 E_s \ll v_0$. In the opposite case $\mu_0 E \gg v_0$ one can neglect time derivatives in (1) and obtain the stationary solution.

The equilibrium electron density n_∞ and the propagation velocity v_0 are defined by conditions of the streamer formation. This stage of the discharge development should be described by essentially nonstationary equations. Their solution depends on the external electric field and parameters of initial "seed". On the quasistationary stage of the streamer propagation the electron density n_∞ can be estimated with the help of the relation

$$r_s \simeq \frac{v_0}{4\pi e\mu_0 n_s}. \tag{12}$$

We assumed here that $n_\infty \simeq n_s$, in agreement with $\rho(0) \sim \rho_s \ll en_s$. Equation (8) allows us to link the charge density ρ_s with E_s. Note that E_s and ρ_s/en_s have logarithmic dependences on the density n_s and radius r_s. Thus, the experimental error in r_s gives rise to a small logarithmic correction to the relative charge ρ_s/en_s.

Let us now compare our results with the experiment. We use the streamer and plasma parameters in the absence of a magnetic field taken from the paper of Dhali and Williams [12] for streamer in N_2 at atmosphere pressure: $v_0 = 2 \cdot 10^8$cm/s, $r_s \simeq 10^{-2}$cm. Substituting these values in Eqs. (11) and (12) one can conclude that $n_s \simeq 3 \cdot 10^{13}\text{cm}^{-3}$, $E_s/E_0 \simeq 0.6$, $\rho_s/en_s \simeq \mu_0 E_s/v_0 \simeq 0.2$. Estimating $\gamma = 0.04$ at $B = 1$T, we have from Eq. (10) for the trajectory curvature radius $R_s \simeq 0.5$cm at $B = 5$T. The experimental value R_s [1] for the same conditions is slightly larger $R_s^{ex} \simeq 1.2$cm. This discrepancy can be explained, e.g., by the growth of the charge density from external region towards the streamer front.

Let us consider the streamer in the limit of an infinitely narrow front in the system of reference where the streamer head is at rest. The electric field E inside the head is sufficiently small so that the ionization process can be safely neglected. The ions are assumed not to be affected by the electromagnetic field, i.e. $N = const$. The corresponding equations are

$$\sum_{kl} \frac{\partial}{\partial \eta_k} \left[n \left(-v_{0k} + \mu_{kl} E_l + \omega \varepsilon_{kl} \eta_l \right) \right] = 0,$$

$$\sum_k \frac{\partial E_k}{\partial \eta_k} = 4\pi \left(N - n \right). \tag{13}$$

For simplicity, the streamer body will beconsidered as a cylinder with radius r_s and the axis directed along η_x-axis. The content of the square brackets in Eq. (13) can be replaced by:

$$\sum_l n \left(-v_{0k} + \mu_{kl} E_l + \omega \varepsilon_{kl} \eta_l \right) = v_0 N \frac{\partial \Phi}{\partial \eta_k}, \tag{14}$$

where the function Φ satisfies the Laplace equation $\Delta\Phi = 0$ inside the streamer body with the boundary conditions

$$\frac{\partial \Phi}{\partial \eta_x} = 1, \frac{\partial \Phi}{\partial \eta_y} = 0 \text{ at } \eta_x = 0; \ \frac{\partial \Phi}{\partial \eta_k} = 0 \text{ at } \eta_y = r_s; \ \Phi \to 0 \text{ at } \eta_x \to \infty. \tag{15}$$

Then, using E_l from (14) and substituting it into the Poisson equation, we obtain for the normalized electron density $\overline{n} \equiv n/N$ the following equation

$$\left(\nu_{ik}\frac{\partial\Phi}{\partial\eta_k}\right)\frac{\partial\overline{n}}{\partial\eta_i} = -\frac{\overline{n}^2\left(\overline{n}-1\right)}{L_0} + \frac{2\gamma\overline{n}}{R_s} \tag{16}$$

with the boundary condition

$$\overline{n}\left(\eta_x = 0, \eta_y = 0\right) = 1 + \Delta\overline{n}, \tag{17}$$

where $\nu_{ik} = \delta_{ik} - \gamma\varepsilon_{ik}$, $L_0 = v_0/(4\pi e\mu_0 N)$ is the characteristic length of the charge relaxation and $\Delta\overline{n} = \rho_s/eN$. Since $\gamma \ll 1$ and $L_0 \simeq r_s \ll R_s$, the second term in (16) may be neglected. In the case of small charge density $e\Delta\overline{n} \ll e\overline{n}$ the equation (16) has a simple analytical solution

$$\overline{n}\left(\eta_x, \eta_y\right) = 1 + \Delta\overline{n}\exp\left(-\frac{s\left(\eta_x, \eta_y\right)}{L_0}\right), \tag{18}$$

where the effective path $s\left(\eta_x, \eta_y\right)$ is defined by a path integration from the point $(0,0)$ to the point $\eta \equiv \left(\eta_x, \eta_y\right)$

$$s\left(\eta_x, \eta_y\right) = \int\limits_{(0,0)}^{\left(\eta_x, \eta_y\right)} \frac{\sum\limits_{ik}\nu_{ik}\frac{\partial\Phi}{\partial\eta_k}d\eta_i}{\sum\limits_{i}\left|\sum\limits_{k}\nu_{ik}\frac{\partial\Phi}{\partial\eta_k}\right|^2}. \tag{19}$$

According to (15), near the front surface $\frac{\partial\Phi}{\partial\eta_i} = \delta_{ix}$, so that to the first order in γ, $s\left(\eta\right) = \eta_x - \gamma\eta_y$ and

$$\overline{n}\left(\eta_x, \eta_y\right) = 1 + \Delta\overline{n}\exp\left(-\frac{\eta_x}{L_0} + \gamma\frac{\eta_y}{L_0}\right). \tag{20}$$

The corresponding electric field is given by

$$E_{\eta_x}\left(\eta_x, \eta_y\right) = E_s\exp\left(-\frac{\eta_x}{L_0} + \gamma\frac{\eta_y}{L_0}\right),$$

$$E_{\eta y}\left(\eta_x, \eta_y\right) = \gamma E_s\exp\left(-\frac{\eta_x}{L_0} + \gamma\frac{\eta_y}{L_0}\right), \tag{21}$$

where $E_s = e\Delta\overline{n}\frac{v_0}{\mu_0}$.

Thus, the electric field E is of the order of E_s ($E \sim E_s$), in η_x-direction in front of the streamer, stimulates its propagation in this direction. It is, therefore, reasonable to suggest that a sufficiently strong electric field component $E_{\eta_y} \sim E_s$, perpendicular to the streamer propagation, will lead to the breakdown of the streamer head and formation of a new streamer deflected along the η_y-direction with respect to the original one. The new streamers obviously will arise only from one side in the plane transverse to $\overrightarrow{B}$. Substituting $\eta_x = 0$, $\eta_y = r_s \simeq L_0$ in (21), we conclude that the condition $E_{\eta_y} \sim E_s$ is fulfilled at $\gamma e^{\gamma} \sim 1$, i.e. $\gamma \simeq 0.6$. This value for the atmosphere discharge in N_2 corresponds to $B = 12$T and agrees well with the experimental result $B^{ex} \simeq 7$T [1].

In conclusion, we have shown that simple model taking into account only the main processes provides a reasonably good description of the streamer discharge in a magnetic field. The streamer head propagation is very similar to the movement of a free charged "particle". Without magnetic field this "particle" moves with a constant velocity $v_0 = const$. In the presence of magnetic field the trajectory has a circular form. Such a simple picture occurs when the external electric field $\mathcal{E}$ is negligible in comparison with the field E_s of the charged streamer head. Nevertheless, the role of the external field $\mathcal{E}$ is very important not only for maintenance of the discharge, but also for the definition of the streamer parameters on the initial stage of its development. Strong electric field $\mathcal{E} \sim E_s \sim E_0$ distorts the circular trajectory making it similar to the trajectory of a charged particle in crossed electric and magnetic fields. This phenomenon was observed in [1].

To estimate the "particle" mass density ρ_m, we compare the expression for the radius,

$$R_s = \frac{\rho_m v_0 c}{\rho_s B},$$

with Eq. (10), which gives us the ratio of the mass and charge densities as

$$\frac{\rho_m}{\rho_s} = \frac{2}{\Delta\overline{n}} \frac{\tau_m}{\tau_{ea}} \frac{m_e}{e}. \tag{22}$$

Due to the large parameter

$$\frac{2}{\Delta\overline{n}} \frac{\tau_m}{\tau_{ea}} \gg 1,$$

the streamer head turns in magnetic field more slowly than a particle with charge density ρ_s and mass density $\rho_m \simeq m_e \rho_s / e$, whose radius does not depend on plasma parameters of the streamer head.

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