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On the theory of polarized Fermi liquid *

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Abstract

Making use the quantum-field theoretical approach we derive the dispersion law for the transverse spin waves in a weakly polarized Fermi liquid at $T = 0$. Along with the dissipationless part inversely proportional to the polarization, it contains also the finite zero-temperature damping. The polarization dependence of both dissipative and reactive part of diffusion constant corresponds to the dependences found earlier by means of kinetic equation with a two-particle collision integral. It is shown that similar derivation for "ferromagnetic Fermi" liquid taking into consideration the divergency of static transverse susceptibility, also leads to the same attenuating spin wave spectrum. Hence, in both cases we deal in fact with spin polarized Fermi liquid but not with isotropic itinerant ferromagnet where the zero temperature attenuation is prohibited by the Goldstone theorem. It demonstrates the troubles of the Fermi liquid formulation of a theory of itinerant ferromagnetic systems.

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The problem of the zero-temperature transverse spin-wave attenuation in spin-polarized Fermi liquid has a long history. The calculations of transverse spin-diffusion coefficient in dilute degenerate Fermi gas with arbitrary polarization was done for the first time in the papers by W. Jeon and W. Mullin [1] where the low temperature saturation of corresponding relaxation time has been established. About the same time A. Meyerovich and K. Musaelyan [2, 3] generalized the Landau derivation [4] of Fermi liquid kinetic equation from microscopic theory to the case of transverse spin kinetics in the polarized Fermi liquid and also came to the same conclusion. A derivation and an exact solution of the kinetic equation in the s-wave scattering approximation for dilute degenerate Fermi gas with arbitrary polarization at $T = 0$ and for a small polarization at $T \neq 0$ have been obtained also in the papers [5] by D. Golosov and A. Ruckenstein. For the treatment of this problem in a Fermi liquid the Matthiessen-type rule arguments and simple relaxation-time approximation for the collision integral have been used [6]. More recently, the derivation of transverse spin dynamics in a spin-polarized Fermi liquid model, from the Landau-Silin kinetic equation with general form of a two-particle collision integral has been performed [7]. The existance of zero-temperature damping of transverse spin waves has been established.

Experimentally the saturation of the transverse spin wave diffusion constant at temperatures about several millikelvin has been registered by the spin-echo technique (see, for instance, [8]). On the other hand, the spin wave experiments demonstrate the behaviour characterized rather by the absence of transverse spin wave damping in the same temperature region [9]. The latter seem to be a confirmation of the point of view of I. Fomin [10] who has argued for the dissipationless form of transversal spin wave spectrum derived from the correction to the system energy due to the gauge transformation into the coordinate system where the magnetization vector is constant. It is worth noting that the similar trick was used earlier for the treatment of one-particle and collective excitations dualism in itinerant ferromagnets by R. Prange and co-workers [11], which is, in our opinion, still unresolved problem (see below). The calculation of the generalized susceptibility coefficient in the expression for the spin current found in [10] has not been performed, just the reference on such calculation [12] in superfluid ^{3}He had been given. Indeed, one can calculate susceptibility using a similar procedure. However, in the case of polarized Fermi liquid one must use the Green functions with the finite imaginary self-energy parts due to collisions between quasiparticles as it was done in [3, 5], that inevitably leeds to the spin waves attenuation.

I. Fomin also used an additional argument in support of absence of attenuation of transversal spin-waves in spin polarized Fermi liquid. This was an analogy with ferromagnetic Fermi liquid where that attenuation in the spin wave spectrum was shown by P. Kondratenko [13] to arise only in the terms of order $\sim k^4$. Indeed, it seems that the space-time evolution of somehow artificially created magnetization in paramagnetic Fermi liquid in the absence of spin-nonconserving interactions will be developed according to the same laws as in isotropic itinerant ferromagnet. In reality this is not true. The conservation of magnetization does not accomplish the dissipationless dispersion of magnons. Even in the inhomogeneously rotating coordinate system, where the magnetization vector is a constant, the quasiparticle distribution function of paramagnetic Fermi liquid is still time and coordinate dependent matrix in the spin space containing odd in momentum off-diagonal part producing the spin current relaxation. In the case of ferromagnet, the corresponding redistribution of the particles over the states with different momenta and spin up and down directions is prevented by the rigidity of many-electron orbital wave function. The last property is not taken into account in the theory of polarized Fermi liquid and all the attempts to discuss the itinerant ferromagnets as sorts of polarized Fermi liquid are incomplete.

The goal of the present article is to reconsider in the frame of microscopic theory the problem of zero temperature spin wave relaxation in spin-polarized Fermi liquid and in "itinerant ferromagnet", as it has been defined in the papers [13, 14]. It is shown that in the both cases the microscopic derivation leads to the same spin wave spectrum with nonzero damping at $T = 0$. If in the former case it just coincides with the conclusion of the papers cited above, in the latter case it contradicts to the results [13, 14] and demonstrates, in our opinion, the failure of the straightforward application of the Fermi liquid concept to the description of itinerant ferromagnetism.

The Landau-type derivation of transverse spin dynamics in a weakly spin-polarized Fermi-liquid from microscopic theory has been performed in the paper [3]. Here we make a similar derivation with the purpose to stress the conditions it needs to be valid, to compare the answer with that obtained from kinetic equation at nonzero temperatures [7], and to juxtapose this with the derivation for "ferromagnetic" Fermi-liquid [13, 14] which we also reproduce afterwards.

As in the original paper by Landau [4], we consider a system of fermions at $T = 0$, with arbitrary short range interaction forces. The presence of polarization means that the particle distribution functions for spin-up and spin-down particles have different Fermi momenta p_+ and p_-. The Green functions near $\mathbf{p} = p_\pm$ and $\varepsilon = \mu_\pm$ have the form

$$G_\pm(\mathbf{p}, \varepsilon) = \frac{a}{\varepsilon - \mu_\pm - v_F(p - p_\pm) + ib(p - p_\pm)|p - p_\pm|}. \qquad (1)$$

We use a weak polarization $v_F(p_+ - p_-) \ll \varepsilon_F$ and also assume that both the Fermi distributions are characterized by the same Landau Fermi liquid parameters. Unlike [15], we introduce here the general form of imaginary part of self-energy [16] which is quadratic function of the difference $(p - p_\pm)$ and changes its sign at $p = p_\pm$, correspondingly. The assumption of small polarization means in particular that G_+ is given by the expression (1) not only near its own Fermi surface $|\mathbf{p}| = p_+$ and $\varepsilon = \mu_+$ but in the whole intervals $p_- < p < p_+$ and $\mu_- < \varepsilon < \mu_+$ and also near the "alien" Fermi surface $|\mathbf{p}| = p_-$ and $\varepsilon = \mu_-$. The same is true for G_-.

In general, the polarization is nonequilibrium, hence $\mu_+ - \mu_- = \Omega - \omega_L$, where $\omega_L = \gamma H_0$ is the Larmor frequency corresponding to the external field H_0 and Ω is the Larmor frequency corresponding to the effective field which would produce the existing polarization , $v_F(p_+ - p_-) = \Omega/(1 + F_0^a)$. [17]

Following Landau, let us write the equation for the vortex function describing scattering of two particles with opposite spin directions and a small transfer of 4-momentum $K = (\mathbf{k}, \omega)$

$$\begin{aligned} &\Gamma(P_1, P_2, K) \qquad (2) \\ = \;& \Gamma_1(P_1, P_2) - \frac{i}{(2\pi)^4} \int \Gamma_1(P_1, Q) G_+(Q) G_-(Q + K) \Gamma(Q, P_2, K) d^4Q \;. \end{aligned}$$

If K is small and polarization is also small, the poles of two Green functions are close to each other. Let us assume that all other quantities in the integrand are slowly varying with respect to Q: their energy and momentum scales of variation are larger than $\max\{\Omega, \omega\}$ and $\max\{\Omega/v_F, k\}$ correspondingly. Then one can perform the integration in (2) at fixed values of $Q = p_0 = (p_+ + p_-)/2,\ \mu = (\mu_+ + \mu_-)/2$ in the arguments of Γ and Γ_1 functions. In other words, one can substitute in (2)

$$\begin{aligned} & G_+(Q) G_-(Q + K) = G_+(\mathbf{q}, \varepsilon) G_-(\mathbf{q} + \mathbf{k}, \varepsilon + \omega) \\ = \;& \frac{2\pi i a^2}{v_F} \delta(\varepsilon - \mu) \delta(|\mathbf{q}| - p_0) \\ \times \;& \frac{\frac{\Omega}{1+F_0^a} + \mathbf{k}\mathbf{v}_F}{\omega - \omega_L + \frac{\Omega F_0^a}{1+F_0^a} + \frac{ib\Omega^2}{v_F^2(1+F_0^a)^2} - \mathbf{k}\mathbf{v}_F + \frac{ib\mathbf{k}\mathbf{v}_F\Omega}{v_F^2(1+F_0^a)}} + \Phi_{\text{reg}}. \qquad (3) \end{aligned}$$

For eliminating Γ_1 from (2) we shall rewrite this equation in the operator form

$$\Gamma = \Gamma_1 - i\Gamma_1(i\Phi + \Phi_{\rm reg})\Gamma, \tag{4}$$

where product is interpreted as integral, and $i\Phi$ denotes the first term from the right-hand-side of eq. (3). In equation (4), we transpose the term involving $\Phi_{\rm reg}$ to the left-hand side, and then apply the operator $(1+i\Gamma_1\Phi_{\rm reg})^{-1}$, obtaining

$$\Gamma = \Gamma^\omega + \Gamma^\omega\Phi\Gamma, \tag{5}$$

where

$$\Gamma^\omega = (1 + i\Gamma_1\Phi_{\rm reg})^{-1}\Gamma_1. \tag{6}$$

As it is known [4], $\Gamma^\omega(\Omega = 0)$ is directly related to the function determining the Fermi liquid interaction,

$$\Gamma^\omega(\Omega = 0) = \Gamma((|\mathbf{k}|/\omega) \to 0, \Omega = 0) = \frac{F_{\mathbf{nn'}}}{a^2 N_0}. \tag{7}$$

At finite Ω the Γ^ω function can be expanded over the polarization as

$$a^2 N_0 \Gamma^\omega = F_{\mathbf{nn'}} + \frac{ib\Omega}{v_F^2(1+F_0^a)} C_{\mathbf{nn'}} + O(\Omega^2). \tag{8}$$

From Eqs. (5) and (8), we come, according to a well known procedure [4], to the kinetic equation

$$\begin{aligned} &\left(\omega - \omega_L + \frac{\Omega F_0^a}{1+F_0^a} + \frac{ib\Omega^2}{v_F^2(1+F_0^a)^2} - \mathbf{kn}v_F + \frac{ib\mathbf{kn}v_F\Omega}{v_F^2(1+F_0^a)}\right)\nu(\mathbf{n}) \\ = &\left(\frac{\Omega}{1+F_0^a} + \mathbf{kn}v_F\right)\int\frac{d\mathbf{n'}}{4\pi}\left(F_{\mathbf{nn'}} + \frac{ib\Omega}{v_F^2(1+F_0^a)}C_{\mathbf{nn'}}\right)\nu(\mathbf{n'}). \end{aligned} \tag{9}$$

We limit ourself to the first two harmonics in the Landau interaction function $F_{\mathbf{nn'}} = F_0^a + (\mathbf{nn'})F_1^a$ and $C_{\mathbf{nn'}} = C_0 + (\mathbf{nn'})C_1$. To obtain the spectrum of the spin waves (see below) obeying the Larmor theorem: the system of spins in a homogeneous magnetic field executes the precessional motion with the Larmor frequency $\omega_L = \gamma H_0$, the coefficient C_0 has to be chosen equal to unity.

Introducing the expansion of the distribution function $\nu(\mathbf{n})$ over spherical harmonics of direction $\mathbf{n} = \mathbf{v}_F/v_F$, one can find from (9) that the ratio of amplitudes of the successive harmonics with $l \geq 1$ is of the order of kv_F/Ω. Hence if this ratio is assumed to be a small parameter one can work with distribution function taken in the form [18] $\nu(\mathbf{n}) = \nu_0 + (\mathbf{n}\hat{\mathbf{k}})\nu_1$. The functions ν_0 and ν_1 obey the following system of linear equations:

$$(\omega - \omega_L)\nu_0 - \frac{kv_F}{3}\left(1 + \frac{F_1^a}{3} - \frac{ib(1 - C_1/3)\Omega}{v_F^2(1 + F_0^a)}\right)\nu_1 = 0, \tag{10}$$

$$-kv_F(1 + F_0^a)\nu_0 + \left(\omega - \omega_L + \frac{\Omega(F_0^a - \frac{F_1^a}{3})}{1 + F_0^a} + \frac{ib(1 - C_1/3)\Omega^2}{v_F^2(1 + F_0^a)^2}\right)\nu_1 = 0. \tag{11}$$

Vanishing of the determinant of this system gives the spin waves dispersion law. At long enough wave-lengths when the dispersive part of $\omega(k)$ dependence is much less than ω_L, we have

$$\omega = \omega_L + (D'' - iD')k^2, \tag{12}$$

where

$$D'' = \frac{v_F^2(1 + F_0^a)(1 + F_1^a/3)}{3\kappa\gamma H} \tag{13}$$

is a reactive part of the diffusion coefficient,

$$D' = \frac{b(1 - C_1/3)(1 + F_0^a)^2}{3\kappa^2} \tag{14}$$

is a dissipative part of the diffusion coefficient, $\kappa = F_0^a - F_1^a/3$, and $H = \Omega/\gamma(1+F_0^a)$ is effective "internal" field corresponding to effective "external" field Ω/γ producing the existing polarization. We derived Eqs. (13) and (14) in the assumption of $\kappa \neq 0$.

The expressions for D' and D'' have been obtained first by the same method by A.Meyerovich and Musaelyan [3]. The former is literally coincides with that found in this paper, the latter has the same parametric dependence but depends in a different way on Fermi liquid parameters. The reason for this is not clear at the moment. These expressions reproduce the corresponding diffusion constants obtained from phenomenological Landau-Silin kinetic equation with two-particle collision integral [7] at arbitrary relation between polarization and temperature if we put in the latters $T = 0$. In particular, D' proves to be polarization independent whereas D'' is inversely proportional to polarization.

Thus, the general microscopic derivation confirms the statement about the existance of zero-temperature spin waves attenuation in polarized Fermi liquid. The value of the dissipative part of spin diffusion D' is determined by the amplitude "b" of the imaginary part of self-energy. Hence it originates from collisions between quasiparticles.

It is worth noting that the inverse proportionality of reactive part diffusion coefficient to polarization is typical for a polarized Fermi liquid. It appears in all the derivations of spin waves dispersion law including Fomin's [10], Prange's [11], and in Stoner model for itinerant ferromagnetism (see, for instance, the book [19]). It has nothing in common with the dispersion law for a ferromagnet which must be proportional to the magnetization as it follows from Landau-Lifshits equation [20] taking into account the domain wall rigidity. The latter is the inherent property of ferromagnet and absent in the paramagnetic polarized Fermi liquid. The domain wall rigidity in itinerant ferromagnet is formed because space-time variations of momentum-dependent off-diagonal, or spin part of quasiparticle distribution function, are blocked up by the inevitable alteration of the orbital part of many-particle electron wave function accompanyed by a huge increase of interaction energy. It is not taken into account in the Fermi liquid theory. From this point the famous Stoner model of ferromagnetism overlooks the most important property of a ferromagnet... So, in our opinion (see also the discussion in [7]) the Fermi liquid theory is applicable to a spin-polarized Fermi liquid but not to itinerant ferromagnets.

There are several known attempts to consider an isotropic itinerant ferromagnetic state as some peculiar type of Fermi liquid. This subject was discussed first phenomenologically by A.A. Abrikosov and I.E. Dzyaloshinskii [21] and then microsopically by P.S. Kondratenko [13]. They did not include in the theory a finite scattering rate between quasiparticles and as result they obtained the dissipationless transvese spin wave dispersion law as it seemed to be in isotropic ferromagnet. The derivation [21] was critisized by C. Herring [22] who pointed out the existance of finite scattering rate and inapplicability of naive Fermi-liquid approach to itinerant ferromagnet (see discussion in [7]). Later I.E. Dzyaloshinskii and P.S. Kondratenko [14] rederived the spin-wave dispersion law in ferromagnets. Starting from the Landau equation for the vertex function for the scattering of two particles with opposite spin direction and a small transfer of 4-momentum, they redefined the product of two Green functions G_+G_- in such a manner that its resonant part was taken equal to zero at $\omega = 0$. This trick gives a possibility to use the $1/k^2$ divergency of transverse static susceptibility, which is an inherent property of degenerate systems and occurs both in an isotropic ferromagnet and in spin polarized paramagnetic Fermi-liquid. The latter of course is true in the absense of interactions violating total magnetization conservation. As in the previous papers [13, 21], the authors of [14] did not introduce a scattering rate in the momentum space between the Fermi surfaces for the particles with opposite spins.

Let us see now what kind of modifications appear if we reproduce the derivation proposed in [14] with the Green functions (1) taking into account the finite quasiparticle scattering rate in the whole interval $p_- < p < p_+$. We discuss first an isotropic ferromagnet at equilibrium $\mu_+ = \mu_-$ in the absence of external field. Following [14] we write:

$$G_+(Q)G_-(Q+K) = G_+(\mathbf{q},\varepsilon)G_-(\mathbf{q}+\mathbf{k},\varepsilon+\omega) \tag{15}$$
$$= \frac{2\pi i a^2}{v_F}\delta(\varepsilon-\mu)\delta(|\mathbf{q}|-p_0)\frac{\omega}{\omega - v_F\Delta + ib\Delta^2 - \mathbf{k}\mathbf{v}_F + \frac{ib\mathbf{k}\mathbf{v}_F\Delta}{v_F}} + \tilde{\Phi}_{\text{reg}},$$

where $\Delta = p_+ - p_-$. Now the Eq. (2) is written as

$$\Gamma = \Gamma_1 - i\Gamma_1(i\tilde{\Phi} + \tilde{\Phi}_{\text{reg}})\Gamma, \tag{16}$$

where $i\tilde{\Phi}$ denotes the first term from the right-hand side of Eq. (15). The equivalent form of this equation is

$$\Gamma = \Gamma^{\mathbf{k}} + \Gamma^{\mathbf{k}}\tilde{\Phi}\Gamma, \tag{17}$$

where

$$\Gamma^{\mathbf{k}} = \Gamma\left(\frac{\omega}{|\mathbf{k}|} \to 0\right) = (1 + i\Gamma_1\tilde{\Phi}_{\text{reg}})^{-1}\Gamma_1. \tag{18}$$

The isotropic part of $\Gamma^{\mathbf{k}}$ is proportional to the transversal susceptibility. Hence it has a singular form [14]

$$\Gamma^{\mathbf{k}} \propto -\frac{1}{N_0(ck)^2}. \tag{19}$$

Here, c is constant with the dimensions of length. It is quite natural to take

$$c \sim \frac{1}{\Delta} \tag{20}$$

such that the divergency (19) disappears in nonpolarized liquid when $\Delta \to 0$. The authors of paper [14] have loss this property by taking $c \sim p_0^{-1}$.

Substitution of Eq. (19) into Eq. (17) gives the transversal spin wave dispersion law

$$\omega = v_F\Delta(ck)^2(1 - \frac{ib\Delta}{v_F}), \tag{21}$$

which proves to be attenuating similar to the polarized Fermi-liquid. One can take into consideration a static external field, by working in the rotating with Larmor frequency coordinate frame that is equivalent to the substitution $\omega \to \omega - \omega_L$ (see also [14]). As a result, we obtain the dispersion law

$$\omega = \omega_L + v_F\Delta(ck)^2(1 - \frac{ib\Delta}{v_F}), \tag{22}$$

which obviously coincides with (12) after taking into account the relation (20).

The attenuating dispersion of transversal spin waves is not surprising because in both cases we deal in fact with spin polarized Fermi liquid but not with isotropic itinerant ferromagnet where the zero temperature attenuation is prohibited by the Goldstone theorem. The Fermi liquid theory leading to the existance of such attenuation is not a correct starting point for the construction of a theory of isotropic itinerant ferromagnetism.

In conclusion, we note that making use of the quantum-field theoretical approach, one can derive the dispersion law for the transverse spin waves in a weakly polarized Fermi liquid at $T = 0$. Along with the dissipationless part inversely proportional to the polarization, it contains also the finite zero-temperature damping. The polarization dependence of both dissipative and reactive part of diffusion constants corresponds to dependences found earlier by means of kinetic equation with two-particle collision integral. The same dispersion law is derived by means of another approach where the divergency of the static transverse susceptibility is taken into consideration. These results obtained for a system of fermions with Fermi liquid type ground state are quite natural for a spin polarized paramagnetic Fermi liquid. On the other hand, in the isotropic itinerant ferromagnet one can expect the dissipationless spin wave spectrum with reactive diffusion constant proportional to magnetization. This demonstrates troubles of the Fermi liquid formulation of a theory of itinerant ferromagnetic systems, which has to operate with an ordered type of ground state.

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