Phononless inelastic transport through a disordered array of quantum wires *

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Abstract

Phononless plasmon-assisted transport through a long disordered array of finite length quantum wires is investigated analytically. Two temperature regimes, the low- and the high-temperature ones with qualitatively different temperature dependencies of thermally activated resistance are identified. The characteristics of plasmon-assisted and phonon-assisted transport mechanisms are compared. Generically strong electron-electron interaction in quantum wires results in a qualitative change of the temperature dependence of thermally activated resistance in comparison to phonon-assisted transport. At high temperatures, the thermally activated resistance is determined by the Luttinger liquid interaction parameter of the wires.

1   Introduction

It is a well established fact that the single-electron transport through a one-dimensional system with random scattering is suppressed at zero temperature and infinitesimally small applied voltage [1]. The physical mechanism of the suppression of transport is known to be the Anderson localization

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phenomenon, which is explained as a constructive interference of initial and back-scattered waves that enhances the return probability after scattering by an impurity potential. Eventually, the constructive interference leads to the localization of a single particle wave function in a one-dimensional disordered system. A natural conclusion follows, that suppression of the coherence of the single particle propagation leads to delocalization and favors transport through the system. Indeed, at finite temperature electrons couple to thermally activated bosonic excitations in the environment (usually phonons), which results in the decoherence of the electron motion and in the thermally activated electron transport [2].

A basic theoretical question remains, whether the thermally activated transport can be induced by electron-electron interactions alone, that is without an external bath of bosonic excitations like phonons. The mechanism of a phononless thermally activated transport consists of dephasing of the electron wave function by electron-electron scattering that results from e-e interactions. In the case of decoherence by interactions, the role of bosonic bath is played by the electron-hole pairs, or charge fluctuations, that are excited thermally or in the result of electron-electron scattering.

Recently the problem of thermally activated transport in a disordered one-dimensional wire with interactions has been addressed in several works [3–5]. In the absence of disorder, a one-dimensional wire with interactions is described by the Tomonaga-Luttinger liquid model. This model allows exact treatment of the low-energy physics in the interacting system using the bosonization technique [6, 7]. At the same time, it is known that the Luttinger liquid is destroyed virtually by any small amount of random scattering [8]. Therefore, in the localized regime that is dominated by disorder, the Luttinger liquid description is inapplicable. The two abovementioned limits suggest two approaches to the problem of thermally activated transport. The first one is based on the description of one-dimensional wire as a disordered Fermi liquid. In that approach, the interactions are treated perturbatively. At low temperatures the conductance is found to be a double exponential function of temperature $\sigma \propto \exp\{\exp[(T\tau)^{-\mu}]\}$ with τ being a mean free time and μ being a parameter of order one [3], instead of the well known thermally activated behavior [2]. The other approach treats the localized system as a pinned charge density wave. The thermally activated transport is described as a propagation of instantons through the system [4, 5]. The theoretical description has then much in common with bosonization. It is applicable for not too weak interactions. That description suggests a thermally activated behavior of conductance similar to the variable range hopping. Most importantly, an external bath of bosonic excitations with

continuous spectrum is necessary to facilitate the transport in the approach of [4], whereas the work [3] predicts a nonvanishing conductance even if the one-dimensional wire is completely isolated from the environment.
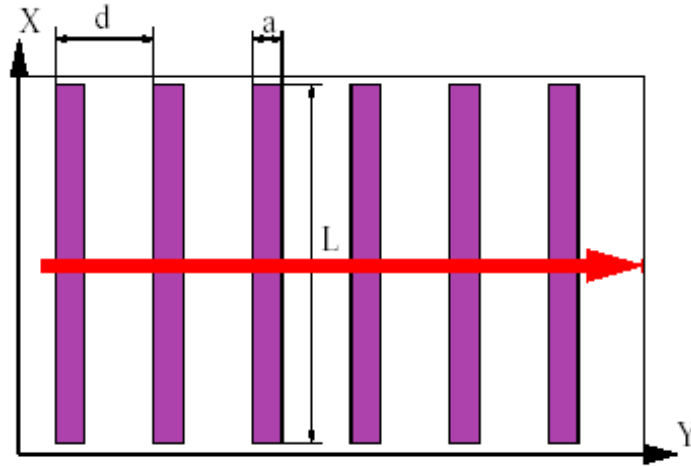

Figure 1: Geometry of the model. The arrow shows the direction of the current.

In the present paper we investigate phononless thermally activated transport through a quasi-one-dimensional system formed by a parallel arrangement of conducting wires (see Fig. 1). Each wire has a finite length L and the transport direction is perpendicular to the wires. We show that charge-density fluctuations (plasmons) in the array can act as the agent promoting thermally activated transport, thus providing the possibility for phononless inelastic transport. As the result of generically strong plasmon-electron coupling in a quantum wire, the features of plasmon- and phonon-assisted transport are qualitatively different. We provide a qualitative explanation of plasmon-assisted transport, identify the transport regimes, where the features of plasmon- and phonon-assisted transport are either similar or substantially different, and derive analytic expressions for the temperature dependence of the thermally activated resistance for a special model of a strongly correlated disordered array of quantum wires. We show that for a special kind of disorder introduced below, the plasmon localization length can noticeably exceed the single particle localization length. Our results apply to the regime when the plasmon localization length exceeds the length of the array.

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Hopping transport in the considered model is of much relevance to a number of experimental setups, including quantum wire arrays in hetero-junctions [9], carbon nanotube films [10], atomic wires on silicon surface [11], and stripe phases [12]. At finite length of constituent wires, such systems represent particular examples of granular arrays, where a one-dimensional wire plays the role of a grain. Considered as a granular array, the array of parallel quantum wires is rather peculiar because of the very long charge relaxation time in a one-dimensional wire. Due to this peculiarity, the theoretical description of thermally activated transport in arrays of long quantum wires requires taking into account the charge dynamics and treatment of the interactions beyond the capacitive model adopted in recent theoretical investigations of transport through disordered granular arrays [5, 13, 14].

2 Theoretical model

The model we formulate below is special, because it combines two seemingly incompatible features: **i)** it is strongly disordered for single electron transport; **ii)** it is much weaker disordered for transport of plasmons.

Consider a one-dimensional array of parallel identical quantum wires of length L and diameter a placed regularly with the interwire distance d, $L \gg d \gg a$. We investigate transport in the direction perpendicular to the wires (see Fig. 1). The spectrum of low-energy plasmons in a single isolated wire is equidistant with energies $E_{i,n} = \frac{\pi v_i}{L} n$, where L is the length of the wire, and v_i is the plasmon velocity along the wire i. For identical wires at regular positions, the intra- and interwire interactions between the charge density fluctuations do not change along the array. Then each localized plasmon level broadens into a plasmon band with truly continuous spectrum, quite analogously to the formation of electronic bands in the tight binding model. The role of hopping in the tight-binding model for plasmons is played by the matrix element of the charge-density interactions between neighboring wires. The formation of plasmon bands is reflected by the dependence of the plasmon velocity along each wire on the wave vector p along the array (that is, perpendicular to the wires) $v_0 \to u(p)$. The particular form of plasmon dispersion depends on details of the interwire interactions, yet the function $u(p)$ should be periodic with a period of one Brillouin zone. That is why we choose the specific form

$$u_p = v_0 + v_1 \cos(\pi p), \ -1 < p \le 1. \tag{1}$$

The chosen form of dispersion can be considered as the first two terms of

the Fourier expansion of some general dispersion law. The plasmon energy within a band centered around the level n is given by $\epsilon_n(p) = \frac{\pi u_p}{L} n$.

2.1 Single particle localization length

Since we are interested in the single particle transport in the direction perpendicular to the wires, we consider in this section only the transverse components of single particle wave functions assuming the factorization of the wave function into the transverse and longitudinal (with respect to the direction of the wires) parts.

Suppose the typical height of potential barriers between neighboring wires is very large. Let us describe the array without interactions by a tight-binding model. Then a single particle wave function has most of its weight inside the wires. We approximate a Wannier wave function localized in a single wire by the form

$$\psi_i(y) = \psi_0 \left\{ \theta\left(y + \frac{a}{2}\right) + \theta\left(\frac{a}{2} - y\right) + e^{-\lambda|y|}\left[\theta\left(-y - \frac{a}{2}\right) + \theta\left(y - \frac{a}{2}\right)\right] \right\}. \tag{2}$$

Here and everywhere further y denotes the coordinate in the direction perpendicular to the wires, that is in the transport direction. Using the normalization condition for $\psi_i(y)$, the value of $|\psi_0|^2$ is found to be

$$|\psi_0|^2 = \frac{\lambda}{a\lambda + 1}. \tag{3}$$

In the absence of disorder and interactions the single particle motion is described by very narrow energy bands with the dispersion law $\epsilon_k = t\cos(kd)$, $\left(-\frac{\pi}{d} < k \leq \frac{\pi}{d}\right)$. With the help of (2), the bandwidth t that equals the hopping energy in the equivalent tight-binding model can be estimated as

$$t \approx \frac{H\lambda d}{a\lambda + 1} e^{-\lambda d}, \tag{4}$$

where λ describes the decay of a single particle wave function inside the barrier of the height H,

$$\lambda \approx \sqrt{\frac{2m}{\hbar^2} H}. \tag{5}$$

Now let us introduce the disorder as random height of the energy barriers between neighboring wires. Such a disorder induces fluctuations of the decay parameter λ thus rendering t random. As a result, the single particle wave functions become localized. In the one-dimensional case the single particle

localization length ξ_1 coincides with the mean free path $l_f = v_F t_f$. The mean free time t_f is, in turn, related to fluctuations of the tunneling barrier height

$$\langle \delta H(n) \delta H(n') \rangle = \frac{1}{2\pi \nu_1 t_f d} \delta_{n,n'}.$$ (6)

Here n and n' denote the numbers of the wires adjacent from the left to the barrier, and we assigned a length scale $1/d$ to the δ-function. ν_1 is a single particle density of states, which is given by $\nu_1 = \frac{1}{td}$ in the center of the band. Now let us establish a connection between the mean free path l_f and the fluctuations $\delta \lambda$ of the decay parameter of the wave function. Using the relation (5), we obtain

$$\delta \lambda = \sqrt{\frac{m}{2\hbar^2 \overline{H}}} \delta H.$$ (7)

Further we solve (6) for τ_f and express δH through $\delta \lambda$ with the help of (7). In the result we obtain

$$l_f = \frac{\overline{\lambda}^4 d^3 e^{-2\overline{\lambda} d}}{8\pi \langle \delta \lambda^2 (a \overline{\lambda} + 1) \rangle}.$$ (8)

Imposing the condition for a single particle to be localized within two nearest neighbor wires $l_f = d$, we obtain for $\langle \delta \lambda^2 \rangle$

$$\langle \delta \lambda^2 \rangle = \frac{\overline{\lambda}^4 d^2 e^{-2\overline{\lambda} d}}{8\pi (a\lambda + 1)}.$$ (9)

2.2 Plasmon localization length

The random height of interwire tunneling barriers affects the interwire charge density interactions through the randomness of the transverse part of the wave functions ψ_i. The matrix element of charge density interactions between neighboring wires with the numbers 1 and 2 (the direct part of

Coulomb interaction) is calculated as

$$V_{12} \propto \int_{-\infty}^{\infty} dy_1 dy_2 \frac{|\psi_1(y_1)|^2 |\psi_2(y_2)|^2}{|d + y^2 - y_1|} = \int_{-a/2}^{a/2} dy_1 dy_2 \frac{|\psi_1(y_1)|^2 |\psi_2(y_2)|^2}{d + y^2 - y_1} +$$

$$\left(\int_{-\infty}^{-a/2} + \int_{a/2}^{\infty} \right) dy_1 \int_{-a/2}^{a/2} dy_2 \frac{e^{-2\lambda|y_1|} |\psi_2(y_2)|^2}{|d + y^2 - y_1|} +$$

$$\int_{-a/2}^{a/2} dy_1 \left(\int_{-\infty}^{-a/2} + \int_{a/2}^{\infty} \right) dy_2 \frac{e^{-2\lambda|y_2|} |\psi_1(y_1)|^2}{|d + y^2 - y_1|} + \qquad (10)$$

$$\left(\int_{-\infty}^{-a/2} + \int_{a/2}^{\infty} \right) dy_1 dy_2 \frac{e^{-2\lambda(|y_1| + |y_2|)}}{|d + y^2 - y_1|}.$$

We took into account the exponential decay of wave functions inside the barrier (for $|y_i| > a/2$). For $\lambda d \gg 1$ the main contribution to the matrix element is given by the first term in (10) that relates to the interaction of charge densities inside the wires. This term results in

$$V_{i,i+1} \sim a^2 |\psi_i|^2 |\psi_{i+1}|^2 / d, \qquad (11)$$

where

$$|\psi_i|^2 = \frac{1}{a} \int_{-a/2}^{a/2} dy |\psi_i(y)|^2. \qquad (12)$$

Note that, being integrated over the wire cross-section, the value $|\psi_i|^2$ is insensitive to random variations of the wave function across the wire. The contributions from other terms in (10) are suppressed as $1/\lambda$. The effect of the random potential barrier on the quantity $|\psi_i|^2$ can be estimated using the normalization condition for the single particle wave function

$$|\psi_i|^2 \left(a + 2 \int_0^{\infty} e^{-2\lambda_i y} dy \right) = 1. \qquad (13)$$

Solving for $|\psi_i|^2$ we obtain

$$|\psi_i|^2 = \frac{\lambda_i}{a\lambda_i + 1}. \qquad (14)$$

The randomness of potential barriers is encoded in the decay factor λ_i. Setting in (14) $\lambda_i = \overline{\lambda} + \delta\lambda_i$, we obtain the fluctuation of $|\psi_i|^2$ in the form

$$\delta|\psi_i|^2 = \frac{\delta\lambda_i}{(a\overline{\lambda} + 1)^2}. \qquad (15)$$

Furthermore, using (11) we estimate the fluctuation of the matrix element of the interaction as

$$\delta V_{i,i+1} \sim \frac{2\overline{\lambda}\delta\lambda_i a^2}{d(a\overline{\lambda}+1)^3}.$$ (16)

In the bosonized form, the density-density interaction between neighboring wires reads

$$H_{i,i+1} = \int_0^L V_{i,i+1} \left(\partial\phi_i(x)\right) \left(\partial\phi_{i+1}(x)\right).$$ (17)

Using the mode expansion of the bosonic fields $\phi_i(x)$, $\phi_{i+1}(x)$ [6]

$$\phi_i(x) = -\sum_q \frac{1}{n_q} \left(e^{-iqx}\hat{b}_{i,q} + e^{iqx}\hat{b}_{i,q}^\dagger\right)$$ (18)

together with the relations $q = \frac{\pi}{L}n_q$, $\omega_q = sq$, where s is the phase velocity of a plasmon along the wire, we rewrite (17) in the form of a hopping Hamiltonian for the bosons represented by operators $\hat{b}_{i,q}, \hat{b}_{i,q}^\dagger$

$$H_{i,i+1} = V_{i,i+1} \sum_q \frac{2\pi\omega_q}{s} \left(\hat{b}_{i,q}^\dagger \hat{b}_{i+1,q} + \hat{b}_{i+1,q}^\dagger \hat{b}_{i,q}\right).$$ (19)

As a result of random interwire interactions, the plasmons become localized in the direction along the array. The plasmon localization length can be evaluated as $\xi_p = u_{gr}\tau_p^\omega$, where τ_p^ω denotes the mean free time of a plasmon mode with frequency ω, and u_{gr} is the group velocity of plasmons. The plasmon mean free time is related to fluctuations of the matrix element of the interwire charge density interactions

$$\left(\frac{2\pi\omega_q}{s}\right)^2 \langle(\delta V_{i,i+1})^2\rangle = \frac{1}{2\pi d\nu_p\tau_p^\omega},$$ (20)

where $\nu_p = \frac{L}{\pi^2 v_1 n}$ is the plasmon density of states in the middle of the band n. In turn, the velocity v_1 is related to the average value of the charge density interactions $\overline{V} = \langle V_{i,i+1}\rangle$. The interaction $\overline{V}_{i,i+1}$ plays the role of the hopping matrix element in the tight binding model for charge-density excitations. Then from the dispersion law of the lowest plasmon band $\epsilon_p = \frac{\pi}{L}(v_0 + v_1\cos(\pi p))$ we infer

$$\overline{V} = \frac{\pi}{L}v_1.$$ (21)

Furthermore, the plasmon group velocity is given by

$$u_{gr} = \left(\frac{1}{d}\right)\frac{d\epsilon_p}{dp} = \frac{\pi^2 d}{L}v_1. \tag{22}$$

By using (11) and (14), the average interaction strength $\overline{V}$ can be expressed as

$$\overline{V} \sim \frac{\lambda^2 a^2}{d(a\lambda + 1)^2}, \tag{23}$$

where $\langle\delta\lambda^2\rangle \ll \lambda^2$ is assumed. Solving (20) for τ_p^ω, expressing the plasmon density of states for the first plasmon band $n = 1$ through the average interaction strength $\overline{V}$ with the help of (21), we obtain the expression for the plasmon mean free time in the form

$$\tau_p^\omega \approx \left(\frac{s}{2\pi\omega}\right)^2 \frac{d(a\overline{\lambda} + 1)^4}{8\langle\delta\lambda^2\rangle a^2}. \tag{24}$$

Now let us evaluate the ratio between the plasmon localization length l_p and the electron localization length d. The condition of strong electron localization on the length d determines the value of $\langle\delta\lambda^2\rangle$ as given by Eq. (9). Substituting this value of $\langle\delta\lambda^2\rangle$ into the expression (24) and multiplying by u_{gr}/d with u_{gr} given by (22), we finally obtain

$$\frac{l_p}{d} \approx \frac{s^2(a\overline{\lambda} + 1)^4}{4\omega^2\overline{\lambda}^2 d^2}e^{2\overline{\lambda}d}. \tag{25}$$

This result agrees qualitatively with the evaluation of plasmon correlation length in a randomly inhomogeneous Luttinger liquid by Gramada and Raikh [15]. Eq. (25) shows that the condition $l_p/d \gg 1$ can be formulated as

$$\overline{\lambda}d \gg \ln\left[\frac{2\omega\overline{\lambda}d}{s(a\overline{\lambda} + 1)^2}\right]. \tag{26}$$

Eq. (26) defines the regime, where the plasmon localization length exceeds very much the electron localization length. If, additionally, l_p exceeds the length of the array in the transport direction (orthogonal to the wires), the plasmon bands can be applied to describe the plasmon spectrum in the array.

3   Resistance of a disordered array

The resistance of a long disordered one-dimensional array is determined by so-called breaks, the junctions between two neighboring wires with exponentially high resistance [16]. Let us denote the energy cost to transfer an

electron over the break as E_a. To facilitate the transport over the break, the energy E_a should be acquired by absorption of a bosonic excitation. For two isolated wires forming the break, the matching condition $E_{i,n} = E_a$ cannot be satisfied for arbitrary E_a because of discreteness of $E_{i,n}$. However, due to the charge-density interwire interaction $H_{int} = \sum_i \sum_{n,n'} V_{i,i+1}^{nn'} \hat{\rho}_i(n) \hat{\rho}_{i+1}(n')$, where n, n' mark the plasmon modes, the energy can be transferred between excitations localized in different wires. In the regime discussed in the previous section the plasmon localization length exceeds the length of the array. Then the plasmon band description of the plasmon spectrum is applicable. Treating the interwire charge density interaction perturbatively, we can write the transition rate caused by the absorption of a plasmon similarly to a transition caused by the absorption of a phonon using the Fermi golden rule [2],

$$
\begin{aligned}
\gamma \quad &\propto \quad \int dp \sum_n \sum_{m,k=0}^{\infty} |V_n(p)| N_B(\epsilon_n(p)) f\left(-E_m\right) \left[1 - f\left(E_a + E_k\right)\right] \times \\
& \delta\left(\epsilon_n(p) - E_a - E_m - E_k\right).
\end{aligned}
\tag{27}
$$

Here $V_n(p)$ is the strength of interwire charge density interaction for the plasmon mode n, $N_B(\epsilon_n(p))$ is the occupation number of the plasmon mode, $f(E_m)$ denotes the Fermi distribution and describes the occupation of the m-th single-particle energy level in the wire, $E_m = \frac{\pi v_0}{L} m$. For narrow plasmon energy bands, the perturbative approach suggests that if the energy E_a lies in the gap between the plasmon bands, hopping over the break is blocked. This suggestion turns out to be wrong because of a conceptual difference between the plasmon and phonon transport mechanisms. Whereas the phonons represent a bath of bosonic excitations that is independent of electrons, the plasmons are "made" of electrons themselves. Consequently, while the electron-phonon interaction can generally be treated perturbatively, the perturbative treatment of plasmons is possible only under special conditions. The applicability of the perturbative treatment of plasmons is determined by the relation of two time scales: the characteristic time of plasmon dynamics t_p and the characteristic time of a single electronic hop t_h. If the Coulomb interaction in a grain is well-screened or plasmons are strongly localized, then t_p is the characteristic relaxation time of a plasmon excitation within a single grain. For $t_p \ll t_h$ the plasmons can be neglected in transport. The description of interactions thus reduces to the capacitive model [5, 13, 14]. For the delocalized undamped plasmons, the time t_p is associated with the formation of an extended in space plasmonic excitation. In that case, $t_p \ll t_h$ correspond to the regime of a strongly nonlinear cou-

pling between plasmons and electrons, and the perturbative treatment of plasmons is incorrect. Plasmons in one-dimensional wires represent a profound example for that regime. In particular, the relation $t_p \ll t_h$ is always fulfilled at the break. For the model considered in this paper, $t_p \sim L/v_1$. As we show below, due to the strong electron-plasmon coupling, the nonlinear effects lead to the creation of plasmon complexes with energies covering the whole spectrum continuously, even though the plasmon bands initially are very narrow. This, in turn, leads to plasmon-assisted transport with a temperature dependence qualitatively different from the case of phonon-assisted transport. In the regime $t_p \gg t_h$, the effective interaction time is limited by t_h. Then the plasmon dynamics is essentially independent of the electron dynamics and, in the case of a continuous plasmon spectrum, the plasmon-assisted transport is quite analogous to the phonon-assisted one.

The resistance of the array is calculated along the lines of Ref. [16]. Let us parametrize the tunneling matrix element between two wires in the form $t_{i,i+1} = \exp(-|y_{i,i+1}|/d)$. The parameter y can be associated with an effective distance between the two wires. This effective distance is random, its distribution follows from the distribution of the heights of potential barriers. Since a break, being a junction with exponentially large resistance, is not shorted by other resistances connected in parallel, we can write the resistance of a break in the form $R_1 = R_0 \exp[2|y_{i,i+1}|/d + f(E_a,T)]$. Here E_a denotes an additional energy to transfer an electron over the break. We remind that the disorder enters the model only as a random distribution of energies E_a and effective distances $y_{i,i+1}$. The function $f(E_a,T)$ accounts for the effect of thermally activated plasmons on the resistance of the break. According to Ref. [16], the probability density $\rho(u)$ for the resistance $R/R_0 = e^u$ is proportional to e^{-gA}, where A is the area in the (y, E_a) phase space that results in the resistance e^u, and g is the linear density of localized one-particle states. The resistance is calculated as $R = R_0 L_y \int_0^\infty du\ e^{u-gA(u)}$, where L_y is the length of the array. Therefore, in order to calculate the resistance of the array in the localized regime, we have to obtain an expression for the resistance of the break R_1. Since the break is not shorted by other resistances, we conclude $R_1 = 1/\sigma_1$, where σ_1 is the conductance of a break. Assume that the break is formed by a junction between the wires with numbers 0 and 1. We take the position of a pinhole connecting the two wires as x. In the linear response approximation the current through the break I_1 is determined by the correlation function [19]

$$X(\tau) = |t_{01}|^2 \left\langle T_\tau \left(\Psi_0(x,\tau)\Psi_1^\dagger(x,\tau)\Psi_1(x,0)\Psi_0^\dagger(x,0) \right) \right\rangle \qquad (28)$$

that characterizes the probability of a single hop over the break, $X_+(\tau) = X(\tau > 0)$ and $X_-(\tau) = X(\tau < 0)$. Here t_{01} is the tunneling matrix element.

A unique feature of the chosen model is the applicability of the bosonized description that allows exact treatment of interactions and hence nonperturbative treatment of plasmons. Precisely, the plasmon dynamics is described by the action

$$S = \int_{-1}^{1} dp \int_0^{\beta} d\tau \int_{-L/2}^{L/2} \frac{dx}{2K_p} \left\{ \frac{1}{u_p} |\partial_\tau \Theta_p|^2 + u_p |\partial_x \Theta_p|^2 \right\}, \qquad (29)$$

representing a finite size generalization of the sliding Luttinger liquid model [18]. The relation of the plasmon velocity u_p and the Luttinger liquid constant K_p with inter- and intrawire interactions has been calculated in [18]. A fermion annihilation operator in the wire n, $\hat{\Psi}_n(x)$, is represented as

$$\hat{\Psi}_n^{\chi}(x) \sim \hat{F}_n^{\chi} \exp \left[-i \int_{-1}^{1} dp \phi_p^{\chi}(x) e^{-i\pi pn} \right], \qquad (30)$$

where $\chi = R, L$ denotes the chirality, $\phi_p^{\chi}(x)$ is a chiral bosonic field, and $\hat{F}_n^{\chi}$ is a Klein factor. The chiral field ϕ_p^{χ} is, in turn, expressed through the field $\Theta_p(x)$ and its dual $\Phi_p(x)$, $\phi_p^{R,L}(x) = (\Theta_p(x) \pm \Phi_p(x)) \sqrt{\pi}$. In the bosonized representation (30), the correlation function $X(\tau)$ factorizes in the correlator of Klein factors and the correlator of bosonic exponents that we denote as $X_b(\tau)$. The time dependence of the Klein factors $F_{n,\chi}(\tau)$ is given by the ground state energy of the wire n that includes the capacitive interaction between the wires. Thus the correlator of the Klein factors is proportional to $e^{-E_a\tau}$. Denoting the correlator of bosonic exponents as

$$X_b^{\chi\chi'}(\tau) = \left\langle T_\tau \left(e^{-i\phi_{0\chi}(0,\tau)} e^{i\phi_{1\chi'}(0,\tau)} e^{-i\phi_{1\chi'}(0,0)} e^{-i\phi_{0\chi}(0,0)} \right) \right\rangle, \qquad (31)$$

we can cast the expression for the current into the form

$$
\begin{aligned}
I_1(V) \;=\; &-\frac{e}{\hbar} \int_{-\infty}^{\infty} dt \times \qquad\qquad\qquad\qquad\qquad\qquad (32) \\
&\sum_{\chi,\chi'=L,R} \left[e^{i(\omega+E_c)t} X_{b-}^{\chi\chi'}(\tau = it + 0) - e^{i(\omega-E_c)t} X_{b+}^{\chi\chi'}(\tau = it + 0) \right].
\end{aligned}
$$

Here $\chi, \chi' = R, L$ denote the chirality of corresponding boson field. We assume the size of the tunneling region along the wire much larger than the Fermi wave-length. Then the terms with equal chiralities, $\chi = \chi'$, give the major contribution to the current (32). Leaving only those terms and

noticing that the contributions form the modes with left and right chiralities to the tunneling current are equal, we can write the conductance of a single junction as

$$\sigma_1(V) = \frac{e^2}{\hbar}\left[\frac{d}{d\omega}X_b(\omega)\Big|_{\omega=-E_c} - \frac{d}{d\omega}X_b(\omega)\Big|_{\omega=E_c}\right].$$ (33)

Substituting the explicit form of the correlator of free bosonic fields, assuming a symmetric form of the dispersion $u(p) = u(-p)$, we finally cast the correlator (31) to the form ($0 \leq \tau \leq \beta$)

$$X_{b+}(\tau) = \exp\left(-\langle\kappa_p S_p(\tau, a)\rangle\right),$$ (34)

where $\kappa_p = K_p + 1/K_p$,

$$S_p(\tau, a) = \sum_{m=0}^{\infty}\left\{\ln\sinh\left[\frac{\pi u_p}{2L}\left((m+1)\beta - \tau\right)\right] + \ln\sinh\left[\frac{\pi u_p}{2L}(m\beta + \tau)\right]\right.$$
$$\left. - \ln\sinh\left[\frac{\pi u_p}{2L}\left((m+1)\beta - a\right)\right] - \ln\sinh\left[\frac{\pi u_p}{2L}(m\beta + a)\right]\right\},$$ (35)

and the average over the plasmon wave vector p is defined by

$$\langle\cdot\rangle_p \equiv \int_0^1 \cdot\left(1 - 2\cos(\pi p) + \cos(2\pi p)\right)dp.$$

For $-\beta < \tau < 0$ the correlator acquires the form

$$X_{b-}(\tau) = \exp\left[-\langle\kappa_p S_p(-\tau, -a)\rangle\right].$$ (36)

In Eqs. (34) and (36), the zero temperature result is given by the terms in (35) with $m = 0$, whereas the terms with $m > 0$ give the temperature correction.

4 Resistance at low temperatures

To analyze the resistance at low temperatures, it is convenient to separate the zero temperature contribution to the current in Eqs. (34) and (35) explicitly. Then the expression for the current can be written as

$$I(eV) \propto \int_{-\infty}^{\infty} d\omega' J_0(eV + \omega')Z(-\omega').$$ (37)

Here $J_0(\omega) \sim \delta(\omega - E_a) - \delta(\omega + E_a)$ defines the zero-temperature current, whereas the influence of thermally activated plasmons is contained in the factor $Z(\omega)$ that is given by the Fourier transform of

$$Z(t) = \exp\left[-\left\langle \kappa_p \sum_{\sigma=\pm 1} \sum_{m=1}^{\infty} \ln\left(1 - e^{-\frac{\pi u_p}{L}(m\beta + i\sigma t)}\right) \right\rangle_p \right]. \qquad (38)$$

Further we assume the coupling constant to be p-independent, $\kappa_p = \kappa$, and use the simplified dispersion law (1). Note that in approximate evaluations it is much more important to keep the p-dependence of the velocity u_p that reflects the formation of plasmon bands than the p-dependence of the coupling constant κ_p. To lowest order in p-dependent terms, the latter just leads to the averaging of the single Luttinger liquid result over the coupling constant. The basic average to be used in subsequent calculations reads

$$\langle e^{-bu_p} \rangle_p = e^{-bv_0}\left[I_0(bv_1) + (1 - \frac{1}{bv_1})I_1(bv_1) \right], \qquad (39)$$

where $b = (m\beta \pm it)\pi/L$, and $I_\nu(z)$ denotes the Bessel function of complex argument. For large times t, (39) gives asymptotically

$$\langle e^{-\frac{\pi u_p}{L}(m\beta \pm it)} \rangle_p \approx \sqrt{\frac{2L}{\pi^2 |v_1|(m\beta \pm it)}} e^{\left[-\frac{\pi}{L}w(m\beta \pm it)\right]}, \qquad (40)$$

where $w = v_0 - v_1$. Despite being obtained for $v_1 < v_0$, (40) is essentially nonperturbative in v_1. The relevant values of the transport time are restricted by the hopping time τ_h. For $v_1|m\beta \pm i\tau_h| < 1$ the correlations giving rise to plasmon bands do not develop, and the short time expansion of (39) has to be used instead of (40). The latter is equivalent to the perturbative treatment of plasmons, leading to a result similar to the phonon mechanism of hopping.

At low temperatures, $T \ll \frac{\pi w}{L}$, the major contribution to $Z(t)$ is given by the term with $m = 1$. Leaving only that term, expanding the logarithms, and substituting (40) in (38), we obtain

$$\begin{aligned} Z(\omega) &\approx \sum_{n,l=0}^{\infty} \frac{2\kappa^{l+n}}{n!l!} \left(\frac{2L}{\pi^2|v_1|}\right)^{\frac{n+l}{2}} e^{-\beta\left(\frac{\pi w}{L}(n+l) + |\omega - \epsilon_{ln}|\right)} \times \\ &\quad |\omega - \epsilon_{ln}|^{\frac{\nu}{2}-1} \frac{\sin\left(\frac{\pi\nu}{2}\right)\Gamma\left(1 - \frac{\nu}{2}\right)}{(2\beta)^{\frac{l+n-\nu}{2}}}, \end{aligned} \qquad (41)$$

where $\epsilon_{ln}(\omega) = \frac{\pi w}{L}(l - n)$, and $\nu = l, n$ for $\omega - \epsilon_{ln} \gtrless 0$, respectively. Each term in (41) describes a thermal excitation of a multiparticle plasmon complex with a continuous density of states. The plasmon complexes described mathematically by (41) form the bath of bosonic excitations that facilitate transport over the break. Higher values of m in (38) would correspond to excitations involving progressively more plasmon modes. The leading terms in (41) at low temperatures are given by $l, n = \overline{0, 1}$. Those terms result in the leading low-temperature contribution to σ_1 in the form

$$\sigma_1 \approx \frac{e^2}{\hbar} \frac{\kappa^2 L \Gamma\left(\frac{1}{2}\right)}{\sqrt{2}\pi^2 |v_1| T} \left(|\beta E_a|^{-\frac{3}{2}} + 2|\beta E_a|^{-\frac{1}{2}} \right) e^{-\beta\left(E_a + \frac{2\pi}{L}w\right)}. \tag{42}$$

Further calculations closely follow Ref. [16] leading to the temperature dependence of the resistance

$$R \propto T^2 \exp\left[\left(\frac{1}{2gdT} + \frac{\pi w}{LT} \right) \right]. \tag{43}$$

For comparison, in the case of phonon-assisted transport in the same model, as well as by a perturbative treatment of the plasmon-electron interactions, the preexponential factor in (43) goes like $T^{1/2}$.

5 Resistance at high temperatures

At high temperatures, when $T \gg \frac{\pi w}{L}$, the temperature broadening exceeds the interlevel separation in a single wire. To analyze the temperature dependence of resistance in that regime, it is convenient to rearrange the sum over m and σ in (35) with the help of the Poisson summation formula

$$S \equiv \sum_{\sigma=\pm 1} \sum_{m=1}^{\infty} \ln\left(1 - e^{-\frac{\pi u_p}{L}(m\beta + i\sigma t)} \right)$$

$$= \sum_{m=-\infty}^{\infty} \ln\left(\frac{\sinh\left[\frac{\pi u_p}{2L}(m\beta + \tau)\right]}{\sinh\left[\frac{\pi u_p}{2L}(m\beta + a)\right]} \right) = \int_{-\infty}^{\infty} dx \ln\left(\frac{\sinh\left[\frac{\pi u_p}{2L}(\beta x + \tau)\right]}{\sinh\left[\frac{\pi u_p}{2L}(\beta x + a)\right]} \right) +$$

$$2\sum_{k=1}^{\infty} \int_{-\infty}^{\infty} dx \cos(2\pi kx) \ln\left(\frac{\sinh\left[\frac{\pi u_p}{2L}(\beta x + \tau)\right]}{\sinh\left[\frac{\pi u_p}{2L}(\beta x + a)\right]} \right). \tag{44}$$

The last line of (44) has been obtained using the Poisson summation formula. After rescaling the integration variable in the integral over x as $y = \frac{\pi u_p}{2L}\beta x$,

we obtain

$$S = \frac{2L}{\pi \beta u_p} \int_{-\infty}^{\infty} dy \ln \left(\frac{\sinh \left[y + \frac{\pi u_p}{2L} \tau \right]}{\sinh \left[y + \frac{\pi u_p}{2L} a \right]} \right) \left[1 + 2 \sum_{k=1}^{\infty} \cos \left(\frac{4kL}{\beta u_p} y \right) \right]. \quad (45)$$

Integrating by parts in each term with a given k, we finally obtain

$$S = \sum_{k=1}^{\infty} S_k, \quad (46)$$

$$S_k = \frac{1}{k} \left[\cos \left(2\pi k \frac{a}{\beta} \right) - \cos \left(2\pi k \frac{\tau}{\beta} \right) \right] \coth \left(\frac{2\pi kL}{\beta u_p} \right). \quad (47)$$

Let us rearrange the sum for S in form of a perturbative expansion around the result for a Luttinger liquid of infinite length. To this end we note, that if we replace $\coth \left(\frac{2\pi kL}{\beta u_p} \right) = 1$ for all k, then the sum over k reads

$$\sum_{k=1}^{\infty} S_k \approx \ln \left| \frac{\sin(\pi \tau/\beta)}{\sin(\pi a/\beta)} \right|, \quad (48)$$

which is the result for the bosonic correlator in infinitely long Luttinger liquid. Therefore, the expression for S can be written in the form

$$S = \log \left| \frac{\sin(\pi \tau/\beta)}{\sin(\pi a/\beta)} \right| + \sum_{k=1}^{\infty} \frac{1}{k} \left[\cos \left(2\pi k \frac{a}{\beta} \right) - \cos \left(2\pi k \frac{\tau}{\beta} \right) \right] \left[\coth \left(\frac{2\pi kL}{\beta u_p} \right) - 1 \right]. \quad (49)$$

Here the first term is the infinite length result, and the sum over k denotes the corrections due to a finite length of the wire L. For large argument of $\coth$, $\left(\frac{2\pi kL}{\beta u_p} \right) \gg 1$, the k-correction decays as $\frac{1}{k} \exp \left(-\frac{4\pi kL}{\beta u_p} \right)$. Further calculation is performed for the zero order term in $1/L$ expansion. Evaluating the Fourier transform of $X_b(\tau) = e^{-\langle \kappa_p S(\tau) \rangle_p}$ and performing the analytical continuation to real frequencies, we obtain

$$\frac{d}{d\omega} X_b(\omega) \approx -i \frac{(2\pi a)^{\kappa}}{2\pi^2 \beta^{\kappa}} \sin(\pi \kappa/2) \Gamma(1 - \kappa) \frac{\Gamma \left(\kappa/2 + i \frac{\beta \omega}{2\pi} \right)}{\Gamma \left(1 - \kappa/2 + i \frac{\beta \omega}{2\pi} \right)} \times$$

$$\left[\psi \left(\kappa/2 + i \frac{\beta \omega}{2\pi} \right) - \psi \left(1 - \kappa/2 + i \frac{\beta \omega}{2\pi} \right) \right]. \quad (50)$$

The conductance of a single junction is then calculated by (33).

For $\frac{\beta E_c}{2\pi} \ll 1$ we get approximately

$$f(T, E_a) \approx \ln\left(\frac{\beta E_a}{\pi} V'(0)\right), \qquad (51)$$

where $V'(0)$ is the derivative $dV(x)/dx|_{x=0}$ of the function

$$V(x) = \frac{\Gamma(\kappa/2 + x)}{\Gamma(1 - \kappa/2 + x)}\left[\psi(\kappa/2 + x) - \psi(1 - \kappa/2 + x)\right]. \qquad (52)$$

One can see that the conductance of a single junction does not fluctuate strongly with the additional energy E_a. Therefore, the resistance of the array is not determined by the resistance of a single break, but is rather given by the average over the resistances of all junctions. The fact that the leading contribution to the conductance at high temperatures is given by the result for infinitely long wires implies that the coherence of the single particle motion is broken already by a single hop between two neighboring wires. The latter justifies the averaging over the all junctions in calculation of the resistance. In the result, the expression for the resistance can be written as a Drude formula with the interaction and temperature dependent mean free time τ_f. The expression for the mean free time can be organized as an expansion in powers of the small parameter $e^{-\frac{4\pi LT}{w}}$. At a typical charging energy $\bar{E}_a > T$, the leading term in the expression for the mean free time is temperature independent, $\tau_f \propto 1/\langle E_a^{2-\kappa}\rangle$. For comparison, phonon-assisted transport in that temperature regime still has a thermally activated character with the preexponential factor $T^{-1/2}$ in (43). At temperatures even larger than the typical charging energy, $\bar{E}_a < T$, τ_f exhibits the power-low temperature dependence typical for transport across a sliding Luttinger liquid, $\tau_f \propto T^{3-\kappa}/\langle E_a\rangle$ [18]. Therefore, at high temperatures, the mean free time is determined by the Luttinger liquid interaction parameter κ. The resistance for phonon-assisted transport in that regime is given by a Drude formula with the logarithmic temperature dependence of the mean free time $\tau_f \propto \ln T$.

6 Conclusion

In conclusion, we demonstrated the possibility of plasmon-assisted inelastic transport in the particular case of a disordered granular array, an array of parallel quantum wires. The chosen specific model of a one-dimensional array enabled us to calculate analytically the preexponential factor of the

thermally activated resistance at low temperatures and investigate differences between the phonon- and plasmon-assisted mechanisms of transport. Due to large charge relaxation time in a wire, the plasmon-electron interaction has to be treated nonperturbatively. In the result, thermally activated resistance has qualitatively different temperature dependence for plasmon-assisted transport as compared to phonon-assisted transport.

Our results show that the capacitive model used for the description of transport in granular arrays [5, 13, 14] is inapplicable for that particar kind of array. The power law dependence of the Drude mean free time in the hight temperature regime is in a general agreement with findings of the recent work [3]. At the same time, we still find a typical thermally activated behavior of resistance at low temperature, although with a modified preexponential factor. Since in the considered regime the plasmons are effectively delocalized, the low temperature regime of [3] with doubly exponential temperature dependence of conductance remains out of the scope of present investigation.

Despite the specific quasi-one-dimensional geometry of the grains in this model, the present investigation is believed to be of a general importance for granular arrays with delocalized or weakly localized plasmons.

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