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Long range coherent manipulation of nuclear spins in quantum Hall ferromagnet

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Abstract

A coherent superposition of many nuclear spin states can be prepared and manipulated via the hyperfine interaction with the electronic spins by varying the Landau level filling factor through the gate voltage in an appropriately designed quantum Hall ferromagnet. During the manipulation periods, the 2D electron system forms spatially large skyrmionic spin textures, where many nuclear spins follow locally the electron spin polarization. The collective spin rotation of a single skyrmion becoming macroscopically massive in the limit of zero Zeeman splitting, may dominate the nuclear spin relaxation and decoherence processes in the quantum well.

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1 Introduction

The emerging fields of quantum information processing and quantum computing (QC) [1] have stimulated recently a flurry of activity in the established fields of atomic and condensed matter physics, approaching fundamental questions, such as the influence of measurement on quantum mechanical systems or the meaning of phase coherence in interacting many particle systems, from a strinkingly new point of view. Experimental realization of QC has been so far successfully achieved, however, only in devices consisting of a few quantum bits (qubits).

The idea presented in this paper should not be considered as a proposal for building any kind of quantum computer. Instead it addresses the general problem of how to store and manipulate a large number of qubits without losing their phase coherence. This is done with respect to a concrete physical system consisting of nuclear spins in semiconducting heterojunctions under the conditions of the odd integer quantum Hall (QH) effect [2]. Our proposal has been motivated by the set of experiments, reported in [3, 4], where the Knight shift, K_S, and the spin lattice relaxation time T_1 of the ^{71}Ga nuclei in GaAs multiple quantum well (MQW) structure under perpendicular magnetic field were detected by means of the optically pumped NMR (OPNMR) technique. The electronic Landau level (LL) filling factor was varied in these experiments by tilting the magnetic field axis with respect to the 2D layers. The Knight shift was found to reduce dramatically as the filling factor was shifted slightly away from $\nu = 1$, indicating that the injection of a single charge into the 2D electron system is followed by reversal of many electronic spins. In the same interval of the filling factor the relaxation time was found to drop by several orders of magnitude with respect to its value in the QH ferromagnetic ground state.

Both effects are considered as strong evidence for the creation of skyrmionic spin texture [5] in the electronic spin distribution as the filling factor shifts slightly away from unity, and indicate the crucial importance of the hyperfine interaction in controlling the nuclear spin dynamics. Since the hyperfine interaction is the dominant coupling of the nuclear spins to their environment they may be exploited as qubits provided the environment, that is the 2D electron gas, is in a nondissipative, coherent quantum state (e.g. like the QH ferromagnetic state at LL filling factor $\nu = 1$ at low temperature [6]). Furthermore, as will be shown below, near $\nu = 1$ it may be possible to manipulate coherently a large number of nuclear spins through the hyperfine interaction with the electronic spin texture by *varying a single parameter*, the LL filling factor, through changes in the gate voltage.

At filling factor $\nu = 1$ the ground state of the 2D electron gas is ferromagnetic even in the limit of zero Zeeman energy [7]. The flipping nuclear spins in this state through the hyperfine interaction is followed by creation of spin excitons [8, 9]. The energy cost of this excitation can be minimized if both the electron and the hole are created at the nuclear position, where the energy gain associated with the e-h Coulomb attraction is exactly compensated by the exchange energy of the hole. Yet, the remaining small Zeeman energy (on the electronic energy scale) is a huge energy gap for the nuclear spins. The extremely long spin-lattice relaxation time observed by Barrett et al. [3, 4] may be due to this energy gap (see below, however). Overcoming the Coulomb attraction by increasing the e-h distance leads to the increase of the exciton transverse momentum. The corresponding excitation energy scales with the Coulomb energy, which is $\sim$ 100K, that is much larger than the Zeeman splitting. The spin exciton spectrum is strongly influenced by long range electrostatic potential fluctuations, which can trap the electron and the hole separately in local potential wells and so reduce, or even completely remove the energy gap [10].

Slightly away from $\nu = 1$ the lowest energy state of the electron gas is a spin texture, in which the average spin distribution is smoothly twisted in space in order to minimize the exchange energy [7]. The size of the twist is determined by the Zeeman energy [11 - 13]. Microscopic calculations based on the Hartree-Fock (HF) approximation for a single, isolated skyrmion [11, 14, 15], have found a family of low energy excitations, with an approximately quadratic relation between the energy and the number of flipped spins, K, which can be associated with the kinetic rotational energy of the entire spin texture around its symmetry axis. However, except for the special case where K is an integer, the spectrum has an excitation gap, which is some fraction of the large Coulomb energy scale. To account for the observed enhancement of the nuclear relaxation rate, the authors have suggested [16] that at filling factor slightly away from $\nu = 1$, where there is a finite density of skyrmions, the ground state is a Skyrme crystal for which the spin waves spectrum is gapless due to the breakdown of the global spin rotation symmetry. This appealing interpretation is hard to reconcile with the latest OPNMR measurements [17]. Based on this data, the many-skyrmion state does not appear consistent with the close packed periodic lattice described in [16]. Instead, it was suggested [17] that the skyrmions' tail is drastically reduced, e.g., due to the effect of disorder potential [18] leading to some kind of spatially inhomogeneous state of nearly independent pinned skyrmions. This conclusion indicates that the problem of spin excitations in a single skyrmion requires further investigation.

It has been recently shown [19] that the excitation gap in the collective rotational spectrum of a single skyrmion goes to zero when the skyrmion radius tends to infinity, and that for the characteristic skyrmion sizes found experimentally the gap is a small fraction of the Zeeman energy scale, rather than of the large Coulomb energy scale, as claimed previously. This low-lying electron spin excitation could strongly influence the nuclear spin relaxation and dephasing processes in the MQW via hyperfine interactions. Our goal in the present paper is therefore to investigate the effect of low-lying electron spin excitations in QH ferromagnet near filling factor $\nu = 1$ on the phase coherence of many nuclear-spin qubits, which are manipulated through the hyperfine interaction by varying the gate voltage.

2 The model

We start our analysis by considering the Hamiltonian for nuclear spins interacting with 2D electron gas in MQW structure

$$\widehat{H} = -\hbar\gamma_n \sum_j \widehat{\mathbf{I}}_j \cdot \mathbf{B}_0 - \hbar\gamma_e \int d^2 r \widehat{\mathbf{S}}\left(\mathbf{r}\right) \cdot \mathbf{B}_0 + \widehat{H}_{ee} + \widehat{H}_{en}, \tag{1}$$

where

$$\widehat{H}_{en} = A \sum_j \widehat{\mathbf{S}}\left(\mathbf{r}_j\right) \cdot \widehat{\mathbf{I}}_j. \tag{2}$$

Here $\widehat{\mathbf{I}}_j$ is the nuclear spin operator located at $\mathbf{r}_j$, $\widehat{\mathbf{S}}\left(\mathbf{r}\right)$ is the electronic spin density operator, $\mathbf{B}_0$ is the external magnetic field, which is assumed to be oriented perpendicular to the 2D electron gas ($\mathbf{B}_0 = B_0 \mathbf{z}$), $\widehat{H}_{ee}$ is the electron-electron interaction, $\gamma_n = g_n \mu_n / \hbar$ and $\gamma_e = g_e \mu_B / \hbar$ are the nuclear and electronic gyromagnetic ratios respectively, and $A = \frac{8\pi}{3} g_n \mu_n g_0 \mu_B \left|u_0\left(0\right)\right|^2$ is the Fermi contact hyperfine coupling constant. In this expression $u_0\left(0\right)$ is the periodic part of the Bloch wavefunction at the nucleus, and g_0 is the g-factor of a free electron. We use the standard normalization $\int_v \left|u_0\left(\mathbf{r}\right)\right|^2 d^3 r = v$, where v is the volume of a unit cell in the crystal.

The manipulation of nuclear spins is carried out through the spin flip-flop processes associated with the 'transverse' part of the interaction Hamiltonian $\widehat{H}_{en}$ (Eq. (2)), i.e. $\frac{1}{2} A \sum_j \left[\widehat{I}_{j,+} \widehat{S}_-\left(\mathbf{r}_j\right) + \widehat{I}_{j,-} \widehat{S}_+\left(\mathbf{r}_j\right)\right]$, where $\widehat{I}_{j,+} = \widehat{I}_{j,x} + i\widehat{I}_{j,y}, \widehat{I}_{j,-} = \widehat{I}_{j,x} - i\widehat{I}_{j,y}$, and $\widehat{S}_+\left(\mathbf{r}\right) = \widehat{\psi}^\dagger_\uparrow\left(\mathbf{r}\right) \widehat{\psi}_\downarrow\left(\mathbf{r}\right)$, $\widehat{S}_-\left(\mathbf{r}\right) = \widehat{\psi}^\dagger_\downarrow\left(\mathbf{r}\right) \widehat{\psi}_\uparrow\left(\mathbf{r}\right)$. Here $\widehat{\psi}_\sigma\left(\mathbf{r}\right), \widehat{\psi}^\dagger_\sigma\left(\mathbf{r}\right)$ are the electron field operators with spin projections $\sigma = \uparrow, \downarrow$.

The strength of the hyperfine coupling constant can be estimated by using the expression

$$K_S \equiv \frac{1}{\hbar} A\langle \widehat{S}_z\left(\mathbf{r}_j\right)\rangle \approx \alpha\left(n_{2D}/2\pi l\right) \tag{3}$$

for the Knight shift at filling factor $\nu = 1$, where $\alpha \equiv A/\hbar$, n_{2D} is the areal density of the 2D electron gas, and l is the QW width. For the ^{31}Ga nucleus (with $g_n \approx .27$) in GaAs $|u_0(0)|^2 \sim 10^4$, and for the parameters characterizing the sample used by Barrett et al. [3], i.e. $l \approx 30$nm, and $n_{2D} = 1.5 \times 10^{11}\text{cm}^{-2}$, one finds $K_S \sim 10^4$Hz, in good agreement with Ref. [3].

In the framework of the model just described, we will now show how, by varying the LL filling factor, a large number of nuclear spins can be prepared in a state appropriate to start quantum computation. A number, n, stored in the memory of a hypothetical quantum computer made of nuclear spins, may be described as a direct product of N pure nuclear spin states

$$|n\rangle = |n_1\rangle \otimes |n_2\rangle \otimes ... \otimes |n_N\rangle\,,$$

where $|n_j\rangle = \sum_{\sigma=\pm 1} \delta_{n_j,\sigma}\,|j,\sigma\rangle$, $\delta_{n_j,\sigma}$ is the Kronecker delta, and $|j,\sigma\rangle$ is a nuclear state with spin projection σ for a nucleus located at $\mathbf{r}_j$. To carry out the quantum computing process, however, a coherent superposition of such products, i.e. $|\psi\rangle = \sum_{n=1}^{N} \alpha_n\,|n\rangle$, should be prepared at the time $t = 0$. This superposition may be represented more transparently for our purposes by the direct product of N mixed spin up and spin down states,

$$|\psi\left(t=0\right)\rangle = \prod_{j=1}^{N} \otimes \left(u_j\,|j,\uparrow\rangle + v_j\,|j,\downarrow\rangle\right)$$

with the normalization $|u_j|^2 + |v_j|^2 = 1$.

While the hyperfine coupling with electron spins is the dominant interaction of the nuclear spin qubits system with its environment, it is only a weak perturbation to the electron spin system. Thus, at a temperature which is much lower than any electronic energy scale in this system, the electronic spins at LL filling factor ν should be in the corresponding ground state,$|0;\nu\rangle$. One may, therefore, construct an effective nuclear spin Hamiltonian by projecting the combined nuclear-electronic spin Hamiltonian, Eq. (1), on the ground electronic state, $|0;\nu\rangle$. The resulting effective nuclear spin Hamiltonian can be written as:

$$\widehat{\mathcal{H}}_n = -\hbar\gamma_n \sum_{j=1}^{N} \widehat{\mathbf{I}}_j \cdot \mathbf{B}_0 + A \sum_{j=1}^{N} \mathbf{S}\left(\mathbf{r}_j\right) \cdot \widehat{\mathbf{I}}_j,$$

where $\mathbf{S}(\mathbf{r}) = \langle 0;\nu| \widehat{\mathbf{S}}(\mathbf{r}) |0;\nu\rangle$ is the expectation value of the electronic spin density in the ground electronic state at filling factor ν.

The corresponding state of the nuclear spin system can be found by considering u_j and v_j as variational parameters, and then minimizing the energy functional $\mathcal{E}_n = \langle\psi| \widehat{\mathcal{H}}_n |\psi\rangle$, with respect to u_j, v_j. As noted above, at $\nu = \nu_0 \neq 1$, $\mathbf{S}(\mathbf{r})$ has nonzero transverse components, associated with the skyrmionic spin texture smoothly varying in space.

A simple calculation shows that

$$\mathcal{E}_n = \frac{1}{2} \sum_j \left\{ \Omega_j \left(|v_j|^2 - |u_j|^2 \right) + \left[A v_j u_j^* S_- (\mathbf{r}_j) + c.c \right] \right\},$$

where $\Omega_j = \gamma_n B_0 - \alpha S_z(\mathbf{r}_j)$ is the local nuclear Zeeman energy. The extremum conditions (subject to the normalization $|u_j|^2 + |v_j|^2 = 1$) are readily solved to yield:

$$|v_j|^2, |u_j|^2 = \frac{1}{2}\left(1 \pm \frac{\hbar\Omega_j}{\varepsilon_j}\right),$$

where

$$\varepsilon_j = \sqrt{\hbar^2\Omega_j^2 + A^2 |S_+(\mathbf{r}_j)|^2}.$$

In this state the nuclear spin polarization $\langle\psi| \widehat{\mathbf{I}}_j |\psi\rangle$ follows the underlying electronic spin texture; the transverse component takes the form

$$I_{j,+} = \langle\psi| \widehat{I}_{j,+} |\psi\rangle = u_j^* v_j = \pm\frac{1}{2} A S_+ (\mathbf{r}_j) / \varepsilon_j, \tag{4}$$

whereas the longitudinal component is

$$I_{j,z} = \langle\psi| \widehat{I}_{j,z} |\psi\rangle = \frac{1}{2}\left(|u_j|^2 - |v_j|^2\right) = \pm\frac{1}{2}\hbar\Omega_j/\varepsilon_j.$$

Thus the nuclear spin distribution follows the distribution of the electronic skyrmion spin texture. The key parameter here is the local mixing parameter

$$\eta_j \equiv (A/\hbar\Omega_j) |S_+(\mathbf{r}_j)| = (2\pi K_S/\Omega_j) \left|\widetilde{S}_+(\mathbf{r}_j)\right|$$

with

$$\widetilde{S}_+(\mathbf{r}_j) \equiv \frac{S_+(\mathbf{r}_j)}{(n_{2D}/l)},$$

which determines the local deviation of the nuclear spins state from a pure ferromagnet. Thus for $\eta_j \ll 1$ the many nuclear spin state is very close to a pure ferromagnet. In the opposite extreme limit, $\eta_j \gg 1$, all individual nuclear spin states are equally probable, i.e. $|v_j|^2 = |u_j|^2 \to 1/2$, and so one generates an ideal starting state for quantum computing [20]. As we will see below, this extremely strong mixing condition is unrealistic. In the intermediate situation, where $\eta_j \sim 1$ almost everywhere, the distributions $|v_j|^2, |u_j|^2$ vary moderately around the mean value 1/2.

The condition for achieving such a desired situation is, therefore, twofolded: (1) the average Knight shift, K_S, should be comparable to the average nuclear Zeeman frequency, Ω, i.e. $(2\pi K_S/\Omega) \sim 1$; and (2) the transverse component of the normalized electronic spin density, $\left|\widetilde{S}_+(\mathbf{r}_j)\right|$, should be of the order one over a large spatial region (namely a region consisting of many nuclear spins). Usually the Knight shift is a small fraction of the NMR frequency, so that the first condition is not easily fulfilled. An exceptional example will be discussed at the end of the paper. The second condition is satisfied by large skyrmionic spin texture (i.e. for sufficiently small effective g-factor).

To place our discussion in the context of the QC paradigm, let us now outline a scenario for coherent manipulation of many nuclear spins in MQW (e.g. by varying the LL filling factor). First of all, the variation of the filling factor is required to be performed without overheating the nuclear spin system. This can be achieved by varying the gate voltage, rather than by tilting the magnetic field direction with respect to the 2D layer, as usually done. If the filling factor is initially tuned at $\nu = \nu_0 \neq 1, \approx 1$, and then kept fixed for a time longer than the (relatively short) relaxation time $T_1(\nu = \nu_0)$ (see Refs. [3, 4]), then the nuclear spins will be settled in the ground state corresponding to the electron gas in the spin texture state at filling factor $\nu = \nu_0$. As shown above, the corresponding nuclear spin distribution, which follows the twisted structure of the electronic spin distribution, may be an appropriate starting point for a QC process. The nuclear spins can return to their spatially uniform (ferromagnetic) state by switching the filling factor back to $\nu = 1$. If the switching is done sufficiently fast (on the time scale $T_1(\nu = \nu_0)$), then the nuclear spins may trap in their twisted state for a long time (i.e. of the order of the large relaxation time $T_1(\nu = 1)$). The next steps may be carried out by varying the filling factor again slightly (but each time differently) away from, and then back to $\nu = 1$, and so manipulating the nuclear spins system via the hyperfine interaction. During each manipulation cycle, when the electronic spins form skyrmionic spin texture, the nuclear spin dynamics is controlled, through the hyperfine interaction, by the low-lying spin excitations characterizing the 2D electron system slightly away from $\nu = 1$.

3 Nuclear spin dynamics

As discussed above, during the manipulation cycle, when the nuclear spins have relatively short relaxation and dephasing times, their dynamics is controlled by the low-lying spin fluctuations of the electron gas through the hyperfine interaction. For a single isolated skyrmion the rigid rotation (by angle φ) of the entire spin texture around its symmetry (Z) axis is a zero mode, which can generate such low energy fluctuations. The generator of this rotation, $\widehat{L}_z$, is the canonical angular momentum conjugate to φ. As discussed in detail in Ref. [5], the eigenvalues of $\widehat{L}_z$ correspond to the total number K of flipped spins in the skyrmion.

It has been recently shown [19] by using a phenomenological approach based on microscopic HF calculation [12, 13] that the angular velocity $\left(\frac{d\varphi}{dt}\right)$ can be written as an effective Larmor frequency for precession of the entire spin texture around the external magnetic field axis,

$$\left(\frac{d\varphi}{dt}\right) \sim \frac{eH}{2M_{scol}c}$$

with an effective mass M_{scol} which diverges with vanishing g-factor as $g^{-5/3}$. For typical experimental values of g it was found that $M_{scol}/m_0 \sim 10^4$.

In addition to the collective rotational motion of the entire spin texture just described, the internal degrees of freedom of the spin texture can also be excited, e.g., as spin waves associated with single electron-hole pair excitations (spin-excitons) [8, 9]. The above consideration shows that for a sufficiently large skyrmion, the energy gap of spin-waves ε_{sp} is much larger than that of the collective rotational spectrum. This separation of energy scales may be expressed explicitly by writing the transverse electron spin density in the form:

$$S_+ (\mathbf{r}, t) \equiv \frac{1}{4\pi} n (\mathbf{r}, t) = \frac{1}{4\pi} \widetilde{n} (\mathbf{r}, t) e^{i\varphi(t)} \tag{5}$$

where $\varphi (t)$ is the instantaneous collective rotation angle and $\widetilde{n} (\mathbf{r}, t)$ stands for all other degrees of freedom in the electronic spin space. It can be derived by expressing the phase of $n (\mathbf{r}, t) = |n (\mathbf{r}, t)| e^{i\theta(\mathbf{r},t)}$ as a Fourier series $\theta (\mathbf{r}, t) = \sum_{\mathbf{k}\neq\mathbf{0}} \theta_{\mathbf{k}} (t) e^{i\mathbf{k}\cdot\mathbf{r}} + \theta_{\mathbf{0}} (t)$ and identifying the uniform term, $\theta_{\mathbf{0}} (t)$, with $\varphi (t)$, so that $\widetilde{n} (\mathbf{r}, t) = |n (\mathbf{r}, t)| \exp \left[\sum_{\mathbf{k}\neq\mathbf{0}} \theta_{\mathbf{k}} (t) e^{i\mathbf{k}\cdot\mathbf{r}}\right]$.

The processes of nuclear spin relaxation and decoherence are reflected in the time dependence of the average $I_{+,-} = \langle \widehat{I}_{+,-} \rangle$, where the brackets $\langle ... \rangle$ stand for the state of the combined system of nuclear and electronic spins (see Ref. [6]). Exploiting the adiabatic approximation, which is valid when the effect of the hyperfine interaction is so weak as to be neglected beyond the leading order, which is the first order in the calculation of the nuclear spin eigen-energies, and the second order in the calculation of relaxation and decoherence. Thus, we have for the transverse component of the nuclear spin located at $\mathbf{r}$, up to the second order of the corresponding perturbation theory [6]:

$$\left[\frac{\partial}{\partial t} + i\Omega\left(\mathbf{r}\right)\right] I_+(\mathbf{r},t) = \\ -\frac{\alpha^2}{4}\int_0^t d\tau \left\langle 0 \left| \left\{ \widehat{S}_+(\mathbf{r},t), \widehat{S}_-(\mathbf{r},\tau) \right\} \right| 0 \right\rangle e^{i\varpi(\tau-t)} I_+(\mathbf{r},t) \tag{6}$$

where the symbol $\{,\}$ stands for anticommutator and the averaging is performed over the ground state $|0\rangle$ of the electronic system. The local NMR frequency $\Omega\left(\mathbf{r}\right)$ corresponds to the unperturbed precession of the nuclear spin in the external static magnetic field (with the frequency $\varpi = \gamma_n B_0$) and the first order correction due to the local hyperfine interaction (the Knight shift), i.e. $\Omega\left(\mathbf{r}\right) = \gamma_n B_0 - \alpha \left\langle 0 \left| \widehat{S}_z\left(\mathbf{r}\right) \right| 0 \right\rangle$. Note that the corresponding correction due to the transverse component of hyperfine field is neglected in Eq. (6). Note also that in the framework of the adiabatic approximation, used in the derivation of Eq. (6), the weak time dependence of the operator $\widehat{I}_+(\tau)$, due to depolarization, is neglected (so that $\widehat{I}_+(\tau) \simeq \widehat{I}_+(t)e^{i\varpi(\tau-t)}$).

The resulting equation, (6), is solved by

$$I_+(\mathbf{r},t) = I_+(\mathbf{r},0)e^{-\Gamma(\mathbf{r},t)-i\Omega(\mathbf{r})t}, \tag{7}$$

where

$$\Gamma\left(\mathbf{r},t\right) = \mathrm{Re}\int_0^t dt'\xi\left(\mathbf{r},t'\right)$$

and

$$\xi\left(\mathbf{r},t\right) = \frac{\alpha^2}{4}\int_0^t d\tau e^{i\varpi(\tau-t)} \left\langle 0 \left| \left\{ \widehat{S}_+(\mathbf{r},t), \widehat{S}_-(\mathbf{r},\tau) \right\} \right| 0 \right\rangle$$

At filling factors slightly away from $\nu = 1$, where the density of skyrmions is small and the interaction between them can be neglected, $\widehat{S}_+(\mathbf{r},t)$ may be written in the form (5) describing a single skyrmion centered at $\mathbf{r} = 0$. On the large time scale relevant to the nuclear spin dynamics of interest here, when the internal degrees of freedom of the spin texture are essentially frozen, it is possible to neglect the time dependence of $\widetilde{n}\left(\mathbf{r},t\right)$ in Eq. (5)

(by writing $\widetilde{n}(\mathbf{r},t) \approx \widetilde{n}(\mathbf{r})$), so that:

$$\xi(\mathbf{r},t) \approx \left(\frac{\alpha}{8\pi}|n(\mathbf{r})|\right)^2 \int_0^t d\tau e^{i\varpi(\tau-t)} \left\langle 0 \left| \left\{ e^{i\widehat{\varphi}(t)}, e^{-i\widehat{\varphi}(\tau)} \right\} \right| 0 \right\rangle,$$

where

$$e^{i\widehat{\varphi}(t)} \equiv e^{it\widehat{H}_{rot}/\hbar} e^{i\varphi} e^{-it\widehat{H}_{rot}/\hbar},$$

and $\widehat{H}_{rot} = \frac{1}{2}U\left(\frac{1}{i}\frac{\partial}{\partial\varphi} - K\right)^2$ [19]. A straightforward algebra yields:

$$e^{i\widehat{\varphi}(t)} = e^{i\varphi} \exp\left\{ i\frac{U}{2\hbar}t\left[1 - 2\left(i\frac{\partial}{\partial\varphi} + K\right)\right]\right\}, \tag{8}$$

so that the correlation function

$$\left\langle 0 \left| \left\{ e^{i\widehat{\varphi}(t)}, e^{-i\widehat{\varphi}(\tau)} \right\} \right| 0 \right\rangle = 2\cos\left[U\delta K(t-\tau)/\hbar\right],$$

where $\delta K \equiv [K] - K$, and $[K]$ is the integer closest to $(K - 1/2)$. Consequently, one finds that

$$\Gamma(\mathbf{r},t) = 2\left(\frac{\alpha}{8\pi}|n(\mathbf{r})|\right)^2 \frac{1 - \cos\left[(U\delta K/\hbar - \varpi)t\right]}{(U\delta K/\hbar - \varpi)^2}. \tag{9}$$

This expression shows that as long as the rotational energy gap $U|\delta K|$ is much larger than the nuclear Zeeman energy $\hbar\varpi$, the off-diagonal element of the nuclear spin density matrix (i.e. the coherence) does not decay but oscillates very quickly (with the frequency $U|\delta K|/\hbar$) between $I_+(\mathbf{r},0)$ and $I_+(\mathbf{r},0)e^{-(A|S_+(\mathbf{r})|/U\delta K)^2}$. It should be stressed that in deriving Eq. (9) the interaction of the electronic system with its environment was completely neglected. This coupling should lead to some energy dissipation, which results in damping of the oscillatory component of $\Gamma(\mathbf{r},t)$, so that for sufficiently long times, $I_+(\mathbf{r},t) \to I_+(\mathbf{r},0)e^{-(A|S_+(\mathbf{r})|/U\delta K)^2}$.

As discussed above, the effective electron g-factor can become locally sufficiently small to make the local skyrmion radius large enough, so that the corresponding rotational energy gap U becomes comparable to the nuclear Zeeman energy $\hbar\varpi$. For such a large skyrmionic spin texture the extremely slow collective spin rotation leads to a complete loss of nuclear spin coherence via the hyperfine coupling. Under this condition the decay is Gaussian, $I_+(\mathbf{r},t) \sim e^{-(\alpha|n(\mathbf{r})|/8\pi)^2 t^2}$, with the characteristic relaxation time

$$T_2 \sim \hbar/A|S_+(\mathbf{r})| = \pi/K_S\left|\widetilde{S}_+(\mathbf{r})\right|,$$

which is of the order of $0.1 - 1$ milliseconds for GaAs MQW. It should be stressed here that the neglect of the first order correction due to the transverse component of the hyperfine field in Eq. (6) results in vanishing of the equilibrium solution $I_+(\mathbf{r}, t \to \infty)$. The present dynamical approach should be therefore modified to take into account this correction in order to describe relaxation to the nonvanishing nuclear spin texture, Eq. (4).

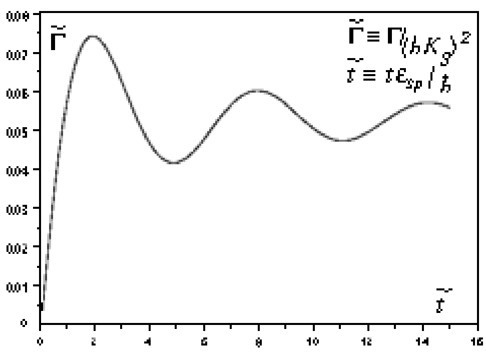

Figure 1: The attenuation exponent of the coherence (off diagonal element of the density matrix) of nuclear spin 1/2 as a function of time in QH ferromagnet for $\varepsilon_C/\varepsilon_{sp} = 32$ (see the text) showing incomplete decoherence due to virtual electronic spin wave excitations.

At filling factor $\nu = 1$, where the number of skyrmions vanishes (note that due to spatial inhomogeneity of the local filling factor some equal number of skyrmions and anti-skyrmions can exist even at $\nu = 1$), the nuclear spin dynamics is controlled by the coupling to the well known gapped spin waves. In the presence of the gap the virtual flip-flop excitations of electronic spin waves via the hyperfine interaction (which are the vacuum quantum fluctuations of the QH ferromagnet) lead to decoherence of the nuclear spin states, i.e. [6]:

$$\Gamma(\mathbf{r}, t) = \Gamma(t) = (hK_S)^2 \int_0^\infty \widetilde{k} d\widetilde{k} e^{-\widetilde{k}^2/2} \frac{1 - \cos([\varepsilon_{ex}(\widetilde{k})/\hbar - \varpi]t)}{[\varepsilon_{ex}(\widetilde{k}) - \hbar\varpi]^2}, \qquad (10)$$

where $\varepsilon_{ex}(\widetilde{k}) \approx \varepsilon_{sp} + \frac{1}{4}\varepsilon_C \widetilde{k}^2$, for $\widetilde{k} = kl_H \ll 1$, and $\varepsilon_C = \sqrt{\pi/2}\left(e^2/\kappa l_H\right)$ is the Coulomb energy. Similar to the case of the collective mode with a large excitations gap discussed below Eq. (9), in the present case the coherence does not decay to zero at any time. In contrast to the effect of the undamped collective mode, however, the presence of a continuous band of spin waves above the Zeeman gap ε_{sp} results in some irreversible loss of coherence. This decoherence occurs on a very short time scale – the precession period of the electronic spin, $2\pi/\omega_{sp}$, whereas for longer times the coherence undergoes damped oscillation (with the frequency ω_{sp}) about a nonzero value [21] (see Fig. 1), that is: $I_+(\mathbf{r},t)e^{i\Omega(\mathbf{r})t} \to I_+(\mathbf{r},0)\exp\left[-2\left(\varepsilon_C/\varepsilon_{sp}\right)\left(hK_S/\varepsilon_{sp}\right)^2\right]$. For $hK_S \ll \varepsilon_{sp}\left(\varepsilon_C/\varepsilon_{sp}\right)^{1/2}$ (e.g. for GaAs MQW $hK_S/\varepsilon_{sp} \sim 10^{-7}$, and $\varepsilon_C/\varepsilon_{sp} \sim 30$ at $H = 10$T), the corresponding decoherence is negligibly small. In actual heterojunctions the electronic Zeeman gap is usually much smaller than the theoretical value. It can be further suppressed by applying pressure [22], so that the situation of gapless spin waves may not be unrealistic experimentally. In this case the integral over $\widetilde{k}$ in Eq. (10) in the long time limit $t \gg \hbar/\varepsilon_C$ acquires the value

$$\Gamma(t) = 2\left(hK_S\right)^2 \int_0^\infty \widetilde{k}d\widetilde{k}e^{-\widetilde{k}^2/2}\frac{\sin^2(\varepsilon_C\widetilde{k}^2t/8\hbar)}{(\varepsilon_C\widetilde{k}^2/4)^2} \to \left(\frac{\left(2\pi\right)^2 K_S^2}{\varepsilon_C/h}\right)t,$$

so that the time decay of coherence is simple exponential

$$I_+(\mathbf{r},t)e^{i\Omega(\mathbf{r})t} \to I_+(\mathbf{r},0)\exp\left(-t/T_2\right),$$

where

$$T_2 = \frac{1}{8\pi^2}\left(\frac{\varepsilon_C/h}{K_S}\right)\left(\frac{1}{K_S}\right).$$

For GaAs MQW this expression yields $T_2 \sim 10^3$ sec., indicating that the long relaxation times observed experimentally in the QH ferromagnetic state can be reasonably explained by a gapless spin exciton spectrum.

4 Conclusion

In this paper it was demonstrated how a coherent superposition of many nuclear spin states can be prepared and manipulated via the hyperfine interaction by varying the LL filling factor in appropriately designed QH ferromagnet. During the manipulation periods the electronic spins form spatially large spin textures where the average spin polarization in the plane perpendicular to the external magnetic field varies smoothly and the individual spins are strongly correlated over large microscopic regions. The nuclear spins coupled to their environment only via the hyperfine interaction with the electron spins follow the changes in the electronic spins system by creating their own spin textures which replicate the electronic ones. This effect is expected to be significant only in very special systems with the strength of hyperfine interaction comparable to the nuclear Zeeman energy. The nuclear spin relaxation and decoherence processes in such states are governed by the coupling to collective spin rotational modes of the entire electronic spin textures, which have a vanishingly small excitation gap in the regions where the local electronic g-factor vanishes.

It turns out that GaAs MQW, despite its remarkable features described above, is not suitable for our purpose. The reason is two-fold:

1) The hyperfine coupling constant in GaAs is much too small to be effective in manipulating nuclear spins in the QW.

2) The nuclear spin dephasing time in QW structures based on GaAs/AlGaAs, is expected to be much smaller than the shortest value of T_1 found in these experiments.

This drawback is due to the fact that all abundant isotopes in this compound (i.e. ^{69}Ga, ^{71}Ga, ^{75}As, all with $I = 3/2$, and ^{27}Al with $I = 5/2$) have non-zero nuclear spins, so that significant dephasing due to dipolar interactions is expected. Indeed, a rough estimate for T_2 for a solid in which each nuclear spin has nearby nuclear spins is in the range of milliseconds [23, 24].

A possible solution for both problems may be found in MQW structures composed of $Si/Si_{1-x}Ge_x$ [25]. The most abundant isotopes of these nuclei have zero nuclear spins so that by purifying the host sample isotopically [25] and then weakly doping with, e.g. ^{31}P donor [26], which has $I = 1/2$, one may reduce the dipolar dephasing to the desired low level. Furthermore, the hyperfine coupling between the conduction electrons and the ^{31}P nucleus in the Si host is strongly enhanced, due to high concentration of electron s-orbitals at the donor nucleus. Thus, a Knight shift of about 30 MHz, which is comparable to the NMR frequency at about 1T, can be obtained for Si:^{31}P [26].

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