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Nonlinear phenomena at the surface of liquid hydrogen *

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Abstract

The results of our recent investigations of the nonlinear phenomena at the charged surface of liquid hydrogen are reported. The reconstruction of the equipotentially charged flat surface is observed resulting in creation of a solitary wave (a hump) at voltages above some critical value. It is demonstrated that excitation of the charged surface by ac electric field at low frequencies results in the turbulent mode in a system of capillary waves. In accordance with the theory of weak wave turbulence the pair correlation function of the surface deviations can be described by the exponential function ω^m. The exponent m changes in magnitude from $m = -3.7 \pm 0.3$ to -2.8 ± 0.2 when the pumping at a single resonant frequency changes to a broadband noise excitation. Measurements are made of the dependence of the boundary frequency ω_b of the upper edge of the inertial range in which the Kolmogorov spectrum is formed on the wave amplitude η_p at the pumping frequency ω_p. It is shown that the obtained data can be well described by a function of the form $\omega_b \propto \eta_p^{4/3}\omega_p^{23/9}$.

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1 Introduction

The main goal of this paper is to present the recent results of our studies [1 - 9] of nonlinear phenomena (static and dynamic) at the charged surface of liquid hydrogen.

The scheme of the experimental setup is shown in Fig. 1. The laser beam is reflected from the charged surface of liquid hydrogen. The dc and ac electric fields are applied between the charged layer and the conical electrode placed above the liquid. The detailed description of the experimental setup is given in the Section 4.

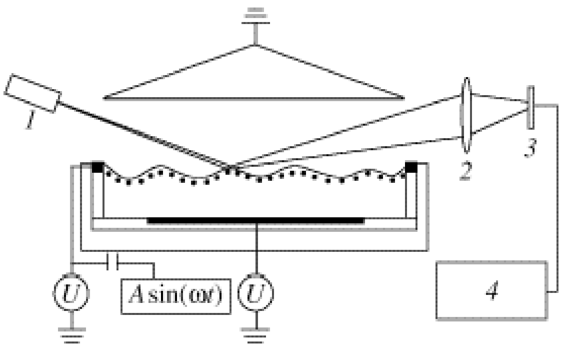

Figure 1: Schematic view of the experimental cell: (1) laser, (2) lens, (3) photodetector, (4) analog-to-digit converter.

In the general case, the dispersion law of surface waves on the flat equipotential charged surface of liquid layer placed between the plates of horizontally arranged capacitor can be written as [10]

$$\omega_k^2 = k\tanh(kh)\left(g + \frac{\alpha k^2}{\rho} - \frac{2kP}{\rho}\coth(kd)\right), \tag{1}$$

where ω_k is the frequency of wave with the wave vector k, h is the thickness of the liquid layer, α is the surface tension, ρ is the density of the liquid, g is the free fall acceleration, d is the distance between the surface and the upper plate of the capacitor, $P = U^2/8\pi d^2$ is the equilibrium pressure of the electric field on the surface, and U is the voltage applied to the capacitor (note that the electric force acting on the charged surface is directed upwards, opposite to the gravitation force).

In the case when the distance from the surface to the control (upper) electrode is smaller than the wavelength ($kd \leq 1$), the dispersion law of the surface waves of a deep liquid (1) simplifies greatly and can be written as

$$\omega_k^2 = k\left(G + \frac{\alpha k^2}{\rho}\right), \tag{2}$$

where $G = g - 2P/\rho\, d$ plays the role of the effective free fall acceleration.

If the voltage applied approaches the critical value $U_{c1} = (4\pi\rho d^3)^{1/2}$, the effective gravitation acceleration G tends to zero and the effective capillary length $\lambda_{\text{eff}} = (\alpha/\rho G)^{1/2} \to \infty$. In this case the surface waves can be considered as purely capillary waves at all k even at the wavelength larger than the capillary length of a neutral liquid $\lambda = \alpha/\rho g^{1/2}$. Hence $\omega_k \propto k^{3/2}$ practically at all k.

From equation (2) it follows that in high electric fields $U > U_{c1}$ where the effective free fall acceleration G becomes negative, a flat charged surface should be unstable against any occasional perturbations with $k \leq \sqrt{\rho|G|/\alpha}$. We have really observed this instability (reconstruction of the initially flat charged surface of liquid hydrogen) experimentally [2]: at the voltages $U > U_{c1}$ a stationary solitary wave (a hump) has been formed at the surface in a cylindrical cell filled with liquid hydrogen. A similar phenomena - formation of a stationary dimple - was observed by Leiderer et al. at the negatively charged surface of liquid helium [2]. It should be mentioned that as the voltage increased above some threshold value U_{c2} - the second critical voltage equal to nearly $1.2\,U_{c1}$ [3, 4], the reconstructed surface of liquid hydrogen lost its stability, and a discharge pulse of the top of the hump was observed (it looked like a geyser). After discharge the surface relaxed to the original flat state and then the process recurred. For this reason it was impossible to study capillary waves at the reconstructed surface of liquid hydrogen at the voltages above $1.2U_{c1}$.

The dispersion curves $\omega(k)$ of oscillations of the charged surface are shown in Fig. 2. The critical voltage in these measurements was equal to $U_{c1} = 1200$ V, the temperature of the measurements was 15 K. As it is seen in Fig. 2, the spectrum is close to $\omega(k) \sim k^{3/2}$, with increasing the voltage the spectrum is softened and no peculiarities have been observed in the field higher than the first critical value. As it was pointed out in the papers [2,3] the observed reconstruction can be discussed in terms of the second order phase transition theory that corresponds to softening of the spectrum of surface waves with raising the external electric field in Fig. 2.

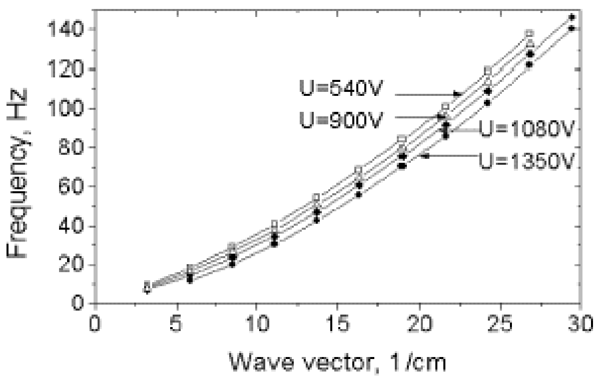

Figure 2: Spectrum of oscillations of the charged surface of liquid hydrogen in a cylindrical cell.

2 Search for turbulence phenomena at the surface of liquid hydrogen

A highly excited state of a system with numerous degrees of freedom, which is characterized by the presence of a directional (in the k space) energy flux, is referred to as turbulent. In the turbulent mode, a system finds itself away from its thermodynamic equilibrium and is characterized by a significant nonlinear interaction of the degrees of freedom, as well as by the dissipation of energy [11]. The nonlinear interaction brings about an effective redistribution of energy between the degrees of freedom (modes). The turbulence may be observed in systems where the excitation frequencies (energy pumping) and dissipation of energy are widely spaced apart at the frequency scale.

Investigations of energy propagation in such systems including capillary waves on the surface of liquid are of great interest from the standpoint of both fundamental nonlinear physics and practical applications. The theory of weak turbulence was developed in the late 1960s (see the monograph [12] and references therein). However, in spite of the large number of experimental investigations of the nonlinear dynamics of surface waves, just a few reports have been published recently of the experimental observations of isotropic spectra of capillary waves on the surface of water [13 - 15], the results of which might be compared with the theoretical predictions.

In this paper we present the results of our recent investigations [5-9] of nonlinear capillary waves on the surface of liquid hydrogen. Liquid hydrogen is a suitable object for experiments in turbulence since it is characterized by a relatively low value of the density ρ and of the kinematic viscosity coefficient ν and by a high value of the coefficient $V \propto (\alpha/\rho^3)^{1/4}$ characterizing nonlinearity of capillary waves. For hydrogen at the temperature $T = 15$ K, it is known that $\alpha = 2.7$ dyn/cm, $\rho = 0.076$ g/cm^3, $\nu = 2.6 \times 10^{-3}$ cm^2/s and $V = 9$ cm$^{3/4}$/sg, and the capillary length $\lambda = 0.19$ cm, while for water at $T = 293$ K $\alpha = 77$ dyn/cm, $\rho = 1.0$ g/cm^3, $\nu = 10^{-2}$ cm^2/s, $V = 3$ cm$^{3/4}$/sg, and $\lambda = 0.28$ cm. This enables us to examine the turbulent mode in liquid hydrogen in a cell with the inner diameter of a few centimeters in a wide frequency range from 10 Hz to 10 kHz. In addition, owing to low density, an external force required to excite oscillations on the surface of liquid hydrogen is several times less than that in the case of water. This fact proved to be decisive in using the procedure in which the waves on the surface are excited by electric forces. The previous experiments have revealed [1] that one can charge the surface of liquid hydrogen with charges injected into the bulk of the liquid, hold the charges in the vicinity of the surface for a long period of time, and excite surface waves using ac electric field. An important advantage of this procedure for the observation of capillary turbulence is the possibility of directly affecting the surface of a liquid by an external force, virtually without acting on the bulk of the liquid, as well as the high degree of isotropism of the exciting force, which enabled one to study the turbulence under well-controlled experimental conditions.

3 Theoretical background

It is known that capillary waves on the surface of a liquid represent an example of nonlinear interacting system. The theory of homogeneous capillary turbulence was described in the paper [16]. It has been demonstrated that an ensemble of weakly interacting capillary waves may be described within a kinetic equation similar to the Boltzmann equation of gas dynamics. The main problem involved in the investigation of wave turbulence is that of finding the law of distribution of the energy of a system of waves with respect to frequency, i.e., the stationary spectrum of the turbulent energy E_ω. The energy E per unit area of liquid surface may be written in the form

$$E = \int \omega_k n_k d\mathbf{k} = \int E_\omega d\omega, \tag{3}$$

where ω_k is the frequency of a wave with the vector $\mathbf{k}$. The capillary wave dispersion law

$$\omega = (\alpha/\rho)^{1/2} k^{3/2} \tag{4}$$

is of the decay type ($\omega'' > 0$) and, therefore, the main contribution to the wave interaction is made by three-wave processes such as the decay of a wave into two with the conservation of the overall wave vector and overall frequency, as well as the reverse process of confluence of two waves into one. A frequency range (inertial range) exists in a system of capillary waves on the surface of a liquid, in which the energy distribution E_ω has the power-like form

$$E_\omega \sim \omega^s.$$

Here s is an exponent that should be estimated from experimental results.

The inertial range is limited from below by the pumping frequency ω_p and at high frequencies by viscous damping. According to the present-day theory [12], when the surface of a liquid is excited at low frequencies belonging to a fairly wide band $\omega_p \pm \Delta\omega$ ("wide-band pumping", $\Delta\omega \approx \omega_p$), a constant energy flux Q towards high frequencies, i.e., direct cascade, sets in the K-space. The theory of homogeneous capillary turbulence predicts the power law dependence on frequency for the wave distribution function n_k and the energy distribution E_ω (Kolmogorov spectrum) within the inertial range, which corresponds to

$$n_k \propto Q^{1/2} \rho^{3/4} \alpha^{-1/4} k^{-17/4} \tag{5}$$

in the k representation.

The steady-state distribution of the surface wave energy in the inertial range may also be equivalently described by the pair correlation function in the Fourier representation

$$I_\omega = \langle |\eta_\omega|^2 \rangle$$

for a deviation of the surface from the planar state $\eta(r,t)$, From the experimental standpoint, it is most convenient to investigate the correlation function I_ω rather than the energy distribution E_ω, because the deviations of the surface from the planar state $\eta(r,t)$ may be measured directly in the experiment. When the surface oscillations are excited in a wide frequency range, the correlation function is predicted by the theory in the form [12]

$$I_\omega = \mathrm{const}\, \omega^{-17/6}. \tag{6}$$

The theoretical prediction of the relation (6) is supported by the results of numerical calculations of the evolution of nonlinear capillary waves performed directly from the first principles using the hydrodynamic equations [17, 18].

In the case of "narrow-band pumping" ($\Delta\omega < \omega_p$), it was demonstrated in the paper [19] that a system of equidistant peaks at frequencies multiple of the pumping frequency was formed on the I_ω curve. The frequency dependence of the peak height is described by a power-like function with the exponent of (-21/6),

$$I_\omega = \mathrm{const}\,\omega^{-21/6}. \tag{7}$$

Note that relations (6) and (7) were derived for systems of capillary waves with a continuous spectrum of wave vectors, i.e., for an idealized infinite surface of liquid. However, under experimental conditions with a limited size of the experimental cell, the $\omega(k)$ spectrum is discrete. This fact must be taken into account in comparing the real correlation function with theoretical predictions. The effect of discreteness decreases with increasing frequency ω, because the resonance width defined by the quality factor increases faster than the distance between the resonances: the spectrum becomes quasicontinuous. In the paper [19] the authors used numerical methods to demonstrate that for discrete systems at a fairly high level of excitation relation (6) is also valid.

As it was mentioned above, the inertial range is limited at high frequencies by the change of energy transfer mechanism from nonlinear wave transformation to viscous damping. The high-frequency edge of the inertial range (boundary frequency) can be defined as a frequency ω_b at which the time τ_v of viscous damping is comparable by the order of magnitude with the characteristic time τ_n of nonlinear interaction (the kinetic time of relaxation in a turbulent wave system), $\tau_v \sim \mathrm{C}\,\tau_n$, where C is some dimensionless constant.

The characteristic time τ_n of nonlinear interaction in a turbulent system is defined by parameters of the liquid, as well as by the capillary wave distribution function $n(\omega)$, and may be estimated as

$$1/\tau_n \sim |V_k|^2 n_k k^2/\omega_k = |V_\omega|^2 n(\omega) \tag{8}$$

where $V_\omega = (\alpha/\rho^{3/2})\omega^{3/2}$ is the coefficient of nonlinearity of capillary waves. The value of τ_n defines the characteristic scale of relaxation times of perturbation over the cascade. It is known [11] that the time of viscous damping of capillary waves decreases with increasing frequency as

$$1/\tau_v = 2\nu\omega^{4/3}(\alpha/\rho)^{2/3}. \tag{9}$$

Relations (8) and (9) enable us to derive the dependence of the wave frequency ω_b on the wave amplitude η_p at the pumping frequency ω_p (narrow pumping)

$$\omega_b \sim \eta_p^{4/3} \omega^{23/9}. \tag{10}$$

The values of the exponents in this equation correspond to the frequency dependence of the correlation function [19]

$$I_\omega \propto \eta_p^2 (\omega/\omega_p)^{-21/6}. \tag{11}$$

Our investigations have shown [5] that a power law dependence on frequency is observed for the correlation function in the frequency range from 100 Hz to 10 kHz when the charged surface of liquid hydrogen is excited by an external periodic electric force at the resonance frequency of the cell. In this case, the exponent in the correlation function was close to -3.7 ± 0.3. And for example, when the surface was excited simultaneously at two resonant frequencies the exponent decreased in magnitude to -2.8 ± 0.2 [6].

The boundary frequency of the upper edge of the inertial range was experimentally determined for the first time in [7]. As the wave amplitude η_p at the pumping frequency ω_p increases, the boundary frequency shifts, by the power law given by Eq. (10), towards high frequencies with the exponent 4/3, as it should be for the case of pumping in a narrow band [9].

4 Experimental procedure

The experiments were performed in an optical cell located in a helium cryostat [8]. The experimental scheme is given in Fig. 1. A plane horizontal capacitor was placed inside the cell. A radioactive plate was located on the bottom capacitor plate. Hydrogen was condensed into a sleeve formed by the bottom capacitor plate and a guard ring 25 mm in diameter and 3 mm high. The layer of liquid was 3 mm thick. The top capacitor plate (a collector 25 mm in diameter) was located at a distance of 4 mm above the surface of the liquid. The temperature of the liquid in the experiments was 15-16 K.

The free surface of the liquid was charged with the aid of the radioactive plate emitting β-electrons into the bulk of the liquid. The electrons emitted by the radioactive plate ionized the thin layer of liquid in the vicinity of this plate. The dc voltage U was applied between the capacitor plates. The sign of the charges forming a quasi-two-dimensional layer below the surface of the liquid was defined by the voltage polarity. In these experiments, the oscillation of a positively charged surface was studied. The metal guard ring installed around the radioactive plate prevented the charges from escaping from under the surface to the container walls. The oscillations of the surface of liquid hydrogen (standing waves) were excited with the aid of the ac voltage applied to the guard ring in addition to the dc voltage at one of the resonant frequencies. The oscillations of the surface of liquid hydrogen were recorded by the variation of the power of a laser beam reflected from the surface. The beam reflected from the oscillating surface was focused by a lens onto a photodetector. The voltage across the photodetector, which was directly proportional to the beam power $P(t)$, was recorded within several seconds by a computer with the aid of a high-speed 12- or 16-bit analog-to-digital converter. We analyzed the frequency spectrum P_ω of the total power of reflected laser beam, which was obtained by Fourier transformation of the $P(t)$ dependence being recorded.

A laser beam 0.5 mm in diameter incident on the surface of the liquid at a grazing angle of about 0.2 rad was used in the experiments. The axes of the light spot ellipse on the surface of the liquid were 2.5 and 0.5 mm. The procedures for excitation of surface oscillation and its recording, as well as the procedure of processing the experimental data, are described in [8]. As it was pointed in this paper, given this size of the light spot, the square of the Fourier amplitude of the measured signal is directly proportional to the correlation function in the frequency representation, $I_\omega \sim P_\omega^2$, at frequencies above 50 Hz.

5 Experimental results

5.1 The effect of the type of pumping on the frequency dependence of the correlation function

As follows from relations (6) and (7), the exponent m in the correlation function $I_\omega \sim \omega^m$ must vary from $m = -21/6$ for the narrow-band pumping to $-17/6$ for the wide-band pumping. The measurement accuracy proved to be sufficient to form a reliable opinion of the variation of the exponent m. Experimental capabilities of the procedure made it possible to obtain and compare the frequency dependencies of correlations functions for three types of excitation of charged surface, namely, at a single resonant frequency, at two resonant frequencies, and by noise in a band covering several resonances.

Fig. 3 gives the frequency dependence of P_ω^2 in the case of excitation of a surface at the resonant frequency of 28 Hz. In the frequency range of 0.2 – 2.0 kHz, the dependence may be well described by a power function. The exponent obtained by averaging over ten measurements is $m = -3.7 \pm 0.3$. For comparison, the solid line in the figure indicates the function $\omega^{-21/6}$, and the dot-and-dash line indicates the function $\omega^{-17/6}$. When the surface was excited at two resonant frequencies, the experimentally obtained P_ω^2 dependencies could be described by a power function with the exponent $m = -2.8 \pm 0.2$, which was close to the predicted value of $m = -17/6$.

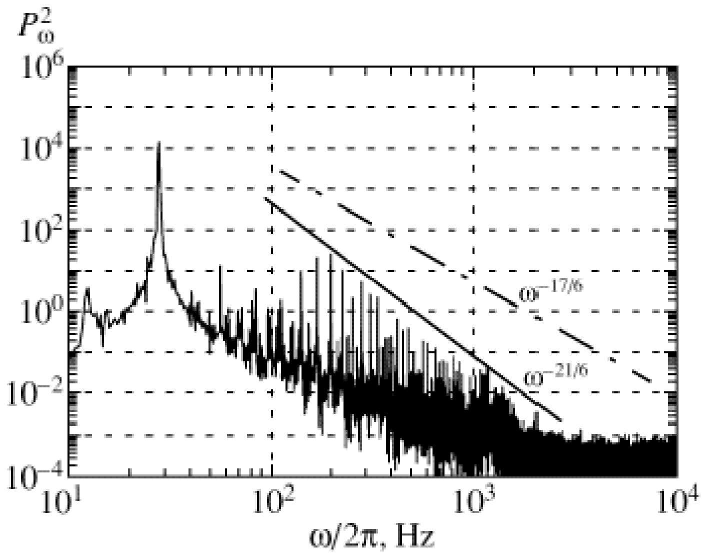

Figure 3: The distribution P_ω^2 in the case of pumping at the frequency of 28 Hz.

When low-frequency noise was used to excite surface oscillation, the P_ω^2 distribution turned out to be close to the predicted dependence given by Eq. (6), as in the case of excitation at two frequencies. Fig. 4 gives the P_ω^2 distribution in the case of surface excitation by noise in the frequency band of approximately 1 to 30 Hz. The solid curve indicates the distribution of the square of Fourier harmonics of the noise voltage applied to the guard ring (expressed in arbitrary units). The dot-and-dash line corresponds to the function $\omega^{-17/6}$. Fig. 4 gives the result obtained by averaging over three files of the distribution. The distribution could be described by a power function of frequency with the exponent $m = -2.8 \pm 0.2$. One can see that the experimentally obtained dependencies turn out to be close to $\omega^{-17/6}$.

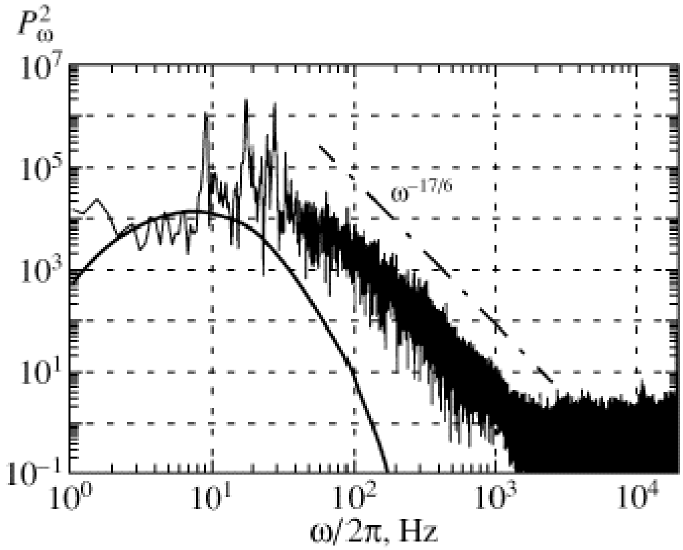

Figure 4: The distribution P^2_ω in the case of pumping by noise at low frequencies. The solid line describes the distribution of the square of Fourier harmonics of the ac voltage applied to the guard ring (in arbitrary units).

5.2 Dependence of the boundary frequency on the wave amplitude at the pumping frequency

As it was observed, the P^2_ω distribution depends on the type of pumping, its amplitude, and the pumping frequency. Fig. 5 gives the frequency dependence of the square of the Fourier amplitude P^2_ω of the $P(t)$ signal, measured during surface excitation at the high frequency $\omega_p = 135$ Hz. The wave amplitude η_p at the pumping frequency was 0.016 mm and the wavelength was $\lambda = 2.3$ mm. The arrow indicates the frequency at which an abrupt variation of the dependence P^2_ω occurs at the edge of the inertial range. For the other amplitudes and pumping frequencies η_p and ω_p, similar results are given in [7, 9]. In Fig. 5, the boundary frequency of the edge of the inertial range is $\omega_b = 4.0 \pm 0.3$ kHz. We have observed that, as the wave amplitude increases, the boundary frequency of the inertial range shifts towards higher frequencies. When the pumping wave amplitude is not high, a cascade consisting of only several harmonics of the pumping frequency ω_p is realized in P^2_ω the spectrum. When the pumping wave amplitude increases, the inertial range is expanded, and the P^2_ω spectrum comes to be made up of tens and even hundreds of harmonics.

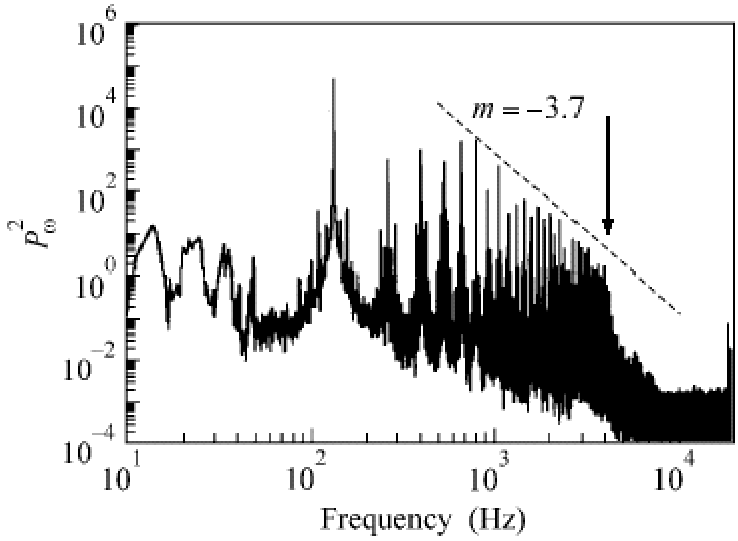

Figure 5: The distribution P_ω^2 with the wave amplitude of 0.016 mm at the pumping frequency of 135 Hz.

Fig. 6 gives three dependencies of the boundary frequency of the edge of the inertial range ω_b on the wave amplitude η_p at the pumping frequencies of 83, 135, and 290 Hz. The ordinates of the points (frequencies) shown in the figure were estimated from the experimentally obtained curves similar to the curves given in Fig. 5. The pumping wave amplitudes were calculated by the known values of ac voltage applied to the guard ring. One can see that the experimentally obtained dependencies $\omega_b(\eta_p)$ may be described by a power function. The solid curves in the figure correspond to the power law dependencies of the boundary frequency of the inertial range ω_b on the amplitude η_p, with the exponent of 4/3, predicted by theoretical considerations, see Eq. (10). For better agreement between the experimental data and the theory, we should add the constant term into the fitting function (it is clear that the boundary frequency ω_b cannot be less than the pumping frequency ω_p). The results of fitting are given in Fig. 6. The constant term turned out to exceed the pumping frequency ω_p by a factor of 2-3.

The amplitude dependence of the boundary frequency ω_b (as given by Eq. (10)) implies the existence of scaling with respect to the pumping frequency ω_p. As it was shown in our paper [9], the experimental points for

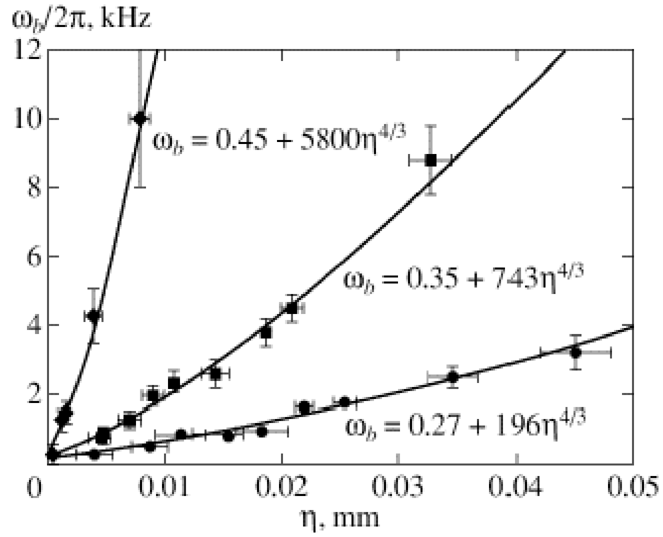

Figure 6: The boundary frequency ω_b as a function of the wave amplitude η_p at the pumping frequencies of 83 (circles), 135 (squares), and 290 (diamonds) Hz.

ω_b, irrespective of the pumping frequency, fit a single straight line in the coordinates $\omega_p/\omega_b^{23/9}$ vs $\eta^{4/3}$ in agreement with the theoretical estimations of Eq. (11). This supports the validity of our assumption of the determining effect of viscosity when estimating the value of the edge frequency ω_b.

6 Conclusion

Investigations of the shape evolution of the equipotentially charged surface of liquid in the external dc electric field demonstrated possibility of observations of the reconstruction phenomenon – formation of a stationary solitary wave at the flat charged surface (a hump in the case of positively charged surface of liquid hydrogen and a dimple in the case of negatively charged surface of superluid He II) at some critical value U_{c1} of the electric field voltage under the conditions of a total screening of the electric field in the bulk of liquid by the surface charge. The critical voltage U_{c1} corresponds to the situation where the applied electric force compensates the gravity force (the effective free fall acceleration tends to zero).

The measurements made at ac electric fields have permitted us to study for the first time propagation and transformation of nonlinear capillary waves at the surface of liquid hydrogen. It is demonstrated that a weak turbulence state (Kolmogorov spectrum) is formed in a system of capillary waves in a wide range of frequencies higher than the driving frequency ω_p. It is observed that the exponent describing the power spectrum of turbulence is changed with changing the spectrum of the driving force. In the case of pumping at a single resonant frequency, the correlation function is described by the power function of frequency with the exponent $m = -3.7 \pm 0.2$, which is close to the predicted value of $m = -21/6$. This corresponds to the stationary spectrum of turbulence $E_\omega \propto \omega^{-13/6}$. In the case of wide-band pumping or excitation of a surface at two resonant frequencies, the observed exponent is $m = -2.8 \pm 0.2$, while the theory gives $m = -17/6$. With this exponent the energy distribution is proportional to $E_\omega \propto \omega^{-3/2}$.

We have also observed the boundary frequency of the inertial range for developed capillary turbulence. It has been found that the inertial range expands towards high frequencies with increasing wave amplitude at the pumping frequency. The wave amplitude dependence of the boundary frequency can be well described by a power function with the exponent of 4/3. The experimental data agree well with the existing theory of weak wave turbulence.

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