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Vibrational and acoustical properties of a liquid drop in the phase-separated ^{3}He-^{4}He fluid with a highly mobile interface *

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Abstract

We have studied the oscillation spectrum and acoustic properties of a liquid drop in a phase-separated fluid when the interfacial dynamics of phase conversion can be described in terms of the kinetic growth coefficient. For a readily mobile interface, i.e., for the growth coefficient comparable with the reciprocal acoustic impedance, anomalous behavior is found in the oscillation spectrum of a drop, as well as in the velocity and absorption of a sound wave propagating through a suspension of drops in the two- phase system. Compared with the known case of two immiscible fluids, the high interface mobility leads to an anomalous softening of the radial drop pulsations and to the frequency- and temperature-dependent behavior of the sound velocity and absorption coefficient in a two-phase suspension.

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1. Below 0.87K a liquid ^{3}He-^{4}He mixture is known to separate into the ^{3}He-concentrated (c) and ^{3}He-dilute (d) phases. Thus, the superfluid-normal interface between the c- and d-phases of a phase-separated liquid mixture can represent a system for studying the dynamic properties of the phase-separation in a supersaturated liquid mixture at low temperatures down to absolute zero. Recently, the interface has been observed to exhibit below 70mK an anomalous behavior in the transmission of sound at frequencies 9, 14 and 32MHz [1]. The sound transmission coefficient at $T \sim$10mK reduces more than 10 times as compared with its high temperature value corresponding to the classical acoustic-mismatch theory.

The amplitude of a transmitted sound wave is directly determined by the boundary conditions for the phase conversion at the interface. For the sound transmission coefficient at normal incidence [2]

$$\tau_{1\to 2} = \frac{2Y_2}{Y_1 + Y_2 + Y_1 Y_2 \xi}, \tag{1}$$

where $Y_{1,2} = \rho_{1,2} c_{1,2}$ are the acoustic impedances given by a product of the density $\rho_{1,2}$ and sound velocity $c_{1,2}$ for the c- and d-phases, respectively. The physical meaning of the kinetic growth coefficient ξ, which vanishes for the two immiscible liquids, can be defined through the relation [3]

$$J = \xi \left(\frac{\rho_1 \rho_2}{\rho_1 - \rho_2} \right)^2 \Delta\mu. \tag{2}$$

Here $\Delta\mu = \mu_1 - \mu_2$ is the difference between μ_1 and μ_2, the chemical potentials per unit mass of the c- and d-phases, respectively. The current J is the mass flow across a unit area of the interface and characterizes the rate of phase conversion as a result of disturbing phase equilibrium due to the sound pressure variation at the interface. In essence, the growth coefficient is a proportionality factor between the growth rate of one of the phases and the phase imbalance.

Thus the anomalous reduction of the sound transmission is an evidence that the interface between the c- and d- phases of a mixture can be highly mobile at sufficiently low temperatures. In fact, the growth coefficient $\xi = \xi(T)$ increases by more than 3 orders of magnitude reaching $Y_2\xi \approx 20 \gg 1$ as $T \sim$10mK [1]. Here we consider the effect of high interface mobility on (a) the spectrum of small oscillations of a c-drop and (b) the velocity and absorption of sound propagating across a suspension of c-drops in a d-phase.

2. Let us consider first the oscillation spectrum of a drop. The problem can be stated as follows. A fluid of density ρ_1 is contained within a sphere of radius R and another fluid phase of density ρ_2 occupies the exterior region confined by a rigid wall of radius $R_b \gg R$ so that the fluid cannot penetrate through the wall. The phase conversion rate at the interface is assumed to be governed by Eq. (2).

In the bulk of both fluids the velocity potentials $\phi_{1,2}$ satisfy the equations

$$\nabla^2\phi_{1,2} - \ddot{\phi}_{1,2}/c_{1,2}^2 = 0. \tag{3}$$

As usual, the interface deformation ζ is supposed to be small and the radius of a drop described by

$$R_\zeta(t) = R + \zeta_l(t)Y_l. \tag{4}$$

Here Y_l is a spherical harmonic function of order $l = 0, 1, 2, ...$ and $\phi_{1,2}(t)$ and $\zeta(t) \propto \exp(-\imath\omega t)$.

The analysis will be restricted to the first order in ζ and so that pressure balance for the disturbed magnitudes of pressure $\delta P_{1,2} = -\rho_{1,2}\dot{\phi}_{1,2}(r,t)$ at the interface gives

$$\delta P_1 - \delta P_2 = \alpha\frac{(l-1)(l+2)}{R^2}\zeta_l, \tag{5}$$

where α is the surface tension and the other factor is a change of the interface curvature [4]. In addition, we involve a continuity of the mass flow across the interface and according to (2) we have

$$\begin{aligned} \rho_1(v_{1,r} - \dot{\zeta}) &= \xi\left(\frac{\rho_1\rho_2}{\rho_1-\rho_2}\right)^2\left(\frac{\delta P_1}{\rho_1} - \frac{\delta P_2}{\rho_2}\right), \\ \rho_2(v_{2,r} - \dot{\zeta}) &= \xi\left(\frac{\rho_1\rho_2}{\rho_1-\rho_2}\right)^2\left(\frac{\delta P_1}{\rho_1} - \frac{\delta P_2}{\rho_2}\right). \end{aligned} \tag{6}$$

Since we consider the total bulk to be closed, the above boundary conditions should be augmented by vanishing the fluid velocity at the wall, i.e., $\nabla\phi_2(r = R_b) = 0$.

Let us first turn to the case of spherically symmetrical pulsations corresponding to $l = 0$. Eliminating ζ in (5) and (6), we get the dispersion equation

$$\omega^2\left(\rho_2\Lambda_2 - \rho_1\Lambda_1\right) - \frac{2\alpha}{R^2} = \imath\omega\rho_1\rho_2\xi\left[\omega^2\Lambda_1\Lambda_2 + \frac{2\alpha}{R^2}\frac{\rho_1\rho_2}{(\rho_1-\rho_2)^2}\left(\frac{\Lambda_2}{\rho_2} - \frac{\Lambda_1}{\rho_1}\right)\right]. \tag{7}$$

The functions $\Lambda_{1,2}$ are ratios of $\phi_{1,2}$ to the velocities $v_{1,2} = \nabla\phi_{1,2}$ taken at the interface $r = R$

$$\begin{aligned} \Lambda_1 &= \frac{R \sin q_1 R}{q_1 R \cos q_1 R - \sin q_1 R} \\ \Lambda_2 &= \frac{-R[\sin q_2(R_b-R) - q_2 R_b \cos q_2(R_b-R)]}{q_2 R[\cos q_2(R_b-R) + q_2 R_b \sin q_2(R_b-R)] + \sin q_2(R_b-R) - q_2 R_b \cos q_2(R_b-R)} \end{aligned}, \quad (8)$$

where the wave vectors are $q_{1,2} = \omega/c_{1,2}$.

To clarify the essentials, let us first compare the two limiting cases, namely, immiscible fluids when $\xi = 0$ and readily miscible ones when $Y_2\xi \gg 1$. In the first case the oscillation spectrum is well known [4]. Provided that both fluids are incompressible, no radial pulsations are possible in the system since any change in the drop volume is strictly forbidden. Finite compressibility results in the appearance of oscillations with the minimum frequency about $\omega \approx 4.5c/R_b$ ($c_1 \approx c_2 \approx c$). The effect of the surface tension is negligible when we do not consider the drop radius smaller than $R_\alpha = 2\alpha/\rho_1 c_1^2$. For our case, $R_\alpha \approx 0.5$Ådue to smallness of $\alpha = 0.0239$ erg/cm^2 [5].

A specific feature of the opposite case $Y_2\xi \gg 1$ is the emergence of a soft mode in the oscillation spectrum, which can lead to an instability of sufficiently small drops. In fact, for the frequencies $\omega \ll c_2/R_b$, Eq. (7) reduces to

$$\omega^2 + \imath \frac{\omega}{Y_2\xi}\frac{c_2}{R} + \frac{2\alpha}{R^3}\frac{\rho_2}{(\rho_1-\rho_2)^2} - \frac{3R}{R_b}\left(\frac{c_2}{R_b}\right)^2 = 0. \quad (9)$$

As it is seen, in the limit $Y_2\xi \to \infty$ only the drops with the size exceeding the critical one

$$R_c = R_b\left(\frac{2\alpha\rho_2}{3(\rho_1-\rho_2)^2 c_2 R_b}\right)^{1/4} \quad (10)$$

are stable against the radial pulsations. The spectrum for drops with $R \gg R_c$ is also enormously softened since $\omega_{min} = (3R/R_b)^{1/2} c_2/R_b \ll c_2/R_b$. The reason for such softening lies in the appearance of a pressure node $\delta P_{1,2} = 0$ at the interface for the infinite growth coefficient, smoothing the pressure variation in the bulk.

The finite magnitude of the growth coefficient brings a damping into the drop pulsations and acts as a stabilizing factor. Provided $R > R_c$, the radial pulsations can be either quasiperiodic with the damping coefficient $(2Y_2\xi)^{-1}c_2/R$ for $2Y_2\xi > (R_b/R)^{3/2} \gg 1$ or aperiodic if $R_b/R < 2Y_2\xi < (R_b/R)^{3/2}$. For small drops with $R < R_c$, the instability remains but a positive imaginary part of ω responsible for the instability strongly decreases. On the whole, if $Y_2\xi \to 0$ and radius $R \approx R_c$,

$$\omega = -\imath Y_2 \xi \, \frac{3R_c^2}{R_b^2} \left(\frac{R^2}{R_c^2} - \frac{R_c^2}{R^2} \right) \frac{c_2}{R_b}. \tag{11}$$

For the high frequency modes of radial pulsations at $\omega \sim c_2/R_b$ or c_1/R, the contribution from the surface tension can be neglected. The finite magnitude of the growth coefficient leads to an additional damping with a maximum at $Y_2\xi \sim 1$ and to a temperature- dependent shift of the pulsation frequency due to $\xi = \xi(T)$.

To gain some insight, we put $c_1 = c_2$ and $\rho_1 = \rho_2$, reducing Eq. (7) to

$$\sin qR_b - qR_b \cos qR_b = \imath Y\xi \sin qR \left[\sin q(R_b - R) - qR_b \cos q(R_b - R) \right]. \tag{12}$$

In the limit $Y\xi \ll 1$ we have a positive shift in frequency

$$\left(\omega - \omega_0 \right) / \omega = -(\imath - Y\xi qR) Y\xi \left(qR_b + 1/qR_b \right) \left(R/R_b \right)^2 , \tag{13}$$

where qR_b is one of the roots of the equation $\tan x = x$. In the opposite case $Y\xi \gg 1$, the shift of the frequency compared with ω_∞ at $Y\xi = \infty$ is negative and we obtain for the frequencies of about c_2/R_b

$$\frac{\omega - \omega_\infty}{\omega} = -\frac{1}{Y\xi} \left(\imath + \frac{1}{Y\xi qR} \right) \frac{1 + (qR_b)^2}{(qR_b)^3} \tag{14}$$

and for the frequencies of about c_1/R

$$\frac{\omega - \omega_\infty}{\omega} = -\frac{1}{Y\xi qR} \left(\imath + \frac{1}{Y\xi} \, \frac{1 + qR_b \tan qR_b}{\tan qR_b - qR_b} \right). \tag{15}$$

Concerning nonspherical pulsations $l \neq 0$, we must note that such type of oscillations is not accompanied by variations of the drop volume. Thus, we can neglect the compressibility of both fluids and consider oscillations in the infinite bulk of the d-phase with $R_b = \infty$. As a result, we find that nonspherical oscillations are described by an equation inherent in oscillatory processes with a single relaxation time

$$\omega^2 - \omega_0^2 = \imath \omega \tau (\omega^2 - \omega_\infty^2). \tag{16}$$

Here τ plays the role of some effective relaxation time for the drop pulsations

$$\tau = \tau_l = \frac{\rho_1 \rho_2 R \xi}{(l+1)\rho_1 + l\rho_2}. \tag{17}$$

The frequency ω_0

$$\omega_0^2 = \frac{(l-1)l(l+1)(l+2)}{(l+1)\rho_1 + l\rho_2} \cdot \frac{\alpha}{R^3} \tag{18}$$

corresponds to the low frequency $\omega\tau \ll 1$ limit and is the well-known oscillation frequency of a liquid drop in a medium of two immiscible fluids considered, e.g., in [6]. In the high frequency limit $\omega_\infty > \omega_0$ and

$$\omega_\infty^2 = (l-1)(l+2)\frac{l\rho_1 + (l+1)\rho_2}{(\rho_1 - \rho_2)^2} \cdot \frac{\alpha}{R^3}. \tag{19}$$

Unlike $l = 0$, oscillations of the shape of a drop in highly miscible fluids prove to be always stable. For $l = 2$, $\omega_\infty/\omega_0 \approx 3.7$ and, if $R = 1\mu$m, $\omega_\infty \approx 3.9$MHz. Since the latter is very close to the experimental conditions in [1], we may expect the high frequency behavior for the oscillations of 1μm-sized c-phase droplets below about 30mK.

3. Here we analyze some acoustic properties of c-drops, namely, scattering of sound wave with a c-drop and the velocity of sound propagating through a suspension of c-drops in the d-phase. In the last case we suppose that the scattering with each c-drop occurs independently of the others. We employ the analogy with optics used for studying acoustic properties in immiscible fluids [7].

It is convenient to introduce the acoustic refraction index

$$n = 1 + \frac{2\pi}{q^2 V}\sum_R f_R(0), \tag{20}$$

where the sum is taken over all c-drops, V is the volume of the system, and $f_R(0)$ is the zero-angle scattering amplitude on a single drop of radius R. The relative change of the sound velocity and the absorption coefficient are given by

$$\frac{\Delta c_2}{c_2} = 1 - Re\, n = -\frac{2\pi}{q^2 V}\sum_R Re\, f_R(0)\ ; \quad \gamma = \frac{\omega}{c} Im\, n. \tag{21}$$

Using the optical theorem [8], we estimate the sound absorption coefficient as

$$\gamma = \frac{1}{2V}\sum_R \sigma_{tot}(R), \tag{22}$$

where $\sigma_{tot}(R)$ is the total cross-section of scattering with a drop of radius R.

Since the scattering amplitude is a sum of partial amplitudes

$$f(0) = \sum_{l=0}^{\infty} (2l+1) f_l(0) \; , \tag{23}$$

we can represent the relative change of the sound velocity and the absorption coefficient as a sum of partial contributions

$$\Delta c_2/c_2 = \sum_{l=0}^{\infty} \Delta c_2^{(l)}/c_2 \; ; \quad \gamma = \sum_{l=0}^{\infty} \gamma^{(l)}. \tag{24}$$

To solve the scattering problem, we use the same Eqs. (3)– (6). The velocity potential outside the drop at $r > R_\varsigma(t)$ should be represented as

$$\phi_2(r,t) = \sum_{l=0}^{\infty} (2l+1)[B_l \jmath_l(q_2 r) + C_l y_l(q_2 r)] \exp(-\imath\omega t), \tag{25}$$

where $\jmath_l(x)$ and $y_l(x)$ are the spherical Bessel functions of the first and second kind, respectively. The partial scattering amplitude is determined by a ratio of C_l/B_l

$$f_l = \frac{\imath}{q_2} \frac{C_l/B_l}{C_l/B_l - \imath} = \frac{\imath}{q_2} \frac{A_l}{A_l - \imath(1 - \omega_l^2/\omega^2)}, \tag{26}$$

where

$$\begin{aligned} A_l \;&=\; \left(\rho_1 q_2 \jmath_l(1) y_l'(2) - \rho_2 q_1 \jmath_l' y_l(2) + \imath\omega\xi\rho_1\rho_2 \jmath_l(1) y_l(2)\right)^{-1} \qquad (27) \\ &\times\; \left\{ \rho_2 q_1 \jmath_l'(1) \jmath_l(2) - \rho_1 q_2 \jmath_l(1) \jmath_l'(2) - \imath\omega\xi\rho_1\rho_2 \jmath_l(1)\jmath_l(2) + \frac{\alpha(l-1)(l+2)}{\omega^2 R^2} \right. \\ &\times\; \left. \left[q_1 q_2 \jmath_l'(1) \jmath_l'(2) - \frac{\imath\omega\xi}{R} \left(\frac{\rho_1\rho_2}{\rho_1 - \rho_2}\right)^2 \left(\frac{q_2}{\rho_1} \jmath_l(1) \jmath_l'(2) - \frac{q_1}{\rho_2} \jmath_l'(1) \jmath_l'(2) \right) \right] \right\} \end{aligned}$$

and the resonance frequency

$$\begin{aligned} \omega_l^2 \;&=\; \left(\alpha(l-1)(l+2)/R^2\right) \qquad (28) \\ &\times\; \left[\rho_1 q_2 \jmath_l(1) y_l'(2) - \rho_2 q_1 \jmath_l'(1) y_l(2) + \imath\omega\xi\rho_1\rho_2 \jmath_l(1) y_l(2)\right]^{-1} \\ &\times\; \left\{ q_1 q_2 \jmath_l'(1) y_l'(2) + \frac{\imath\omega\xi}{R} \left(\frac{\rho_1\rho_2}{\rho_1 - \rho_2}\right)^2 \left(\frac{q_1}{\rho_1} \jmath_l'(1) y_l(2) - \frac{q_2}{\rho_1} \jmath_l(1) y_l'(2) \right) \right\}. \end{aligned}$$

The argument in $\jmath_l$ and y_l is either $q_1 R$ or $q_2 R$.

Since the general expressions are rather cumbersome, we concentrate on the wavelength limit $q_{1,2}R \ll 1$. In this limit the main contribution to the total scattering amplitude comes from the lowest harmonics, $l = 0$ and $l = 1$ if $\xi = 0$ and from $l = 0$ alone if $\xi = \infty$. The partial $l = 0$ amplitude is given by

$$f_0 = -\frac{R}{3}(q_2R)^2 \frac{(1-\rho_2c_2^2/\rho_1c_1^2) + 3(1-\omega_0^2/\omega^2)(Y_2\xi)^2 - 3\imath Y_2\xi/(q_2R)}{1+(1-\omega_0^2/\omega^2)^2(\omega R\rho_2\xi)^2}, \tag{29}$$

where ω_0 is the normal frequency of the radial pulsations determined by Eq. (9). The total scattering cross-section $\sigma_{tot}^{(0)} = \sigma_{el}^{(0)} + \sigma_{in}^{(0)}$ consists of two terms: the elastic one

$$\sigma_{el}^{(0)} = 4\pi R^2\left(\frac{\omega R}{3c_2}\right)^2 \frac{(1-\rho_2c_2^2/\rho_1c_1^2)^2(\omega R/c_2)^2 + 9(Y_2\xi)^2}{1+(1-\omega_0^2/\omega^2)^2(\omega R\rho_2\xi)^2} \tag{30}$$

and the inelastic one meaning an additional absorption of sound at $0 < \xi < \infty$:

$$\sigma_{in}^{(0)} = 4\pi R^2 \frac{Y_2\xi}{1+(1-\omega_0^2/\omega^2)^2(\omega R\rho_2\xi)^2}. \tag{31}$$

For $\xi = 0$, Eqs. (29)–(31) reduce to the well-known case of two immiscible liquids [4]. The assumption $\xi \neq 0$ leads to an enhancement of sound scattering, making it temperature-dependent due to $\xi = \xi(T)$. At large ξ satisfying $Y_2\xi \gg (q_2R)^{-1} \gg 1$, the cross-section tends to

$$\sigma = 4\pi R^2\omega^4/(\omega^2-\omega_0^2)^2 \tag{32}$$

grading for $\omega \gg \omega_0$ into the cross-section $\sigma = 4\pi R^2$ corresponding to the scattering by an impermeable sphere of radius R. The statement becomes clear if we take into account that the sound wave perturbation cannot penetrate into a drop for $\xi = \infty$.

The contribution with $l = 0$ to the sound velocity variation caused by c-drops is

$$\frac{\Delta c_2^{(0)}}{c_2} = \frac{1}{2V}\sum_R \frac{4\pi R^3}{3}\cdot\frac{(1-\rho_2c_2^2/\rho_1c_1^2) + 3(1-\omega_0^2/\omega^2)(Y_2\xi)^2}{1+(1-\omega_0^2/\omega^2)^2(\omega R\rho_2\xi)^2}. \tag{33}$$

At $Y_2\xi > \mid 1-\rho_2c_2^2/\rho_1c_1^2 \mid \sim 1$, $\Delta c_2^{(0)}/c_2$ becomes temperature- and frequency-dependent. The strong effect gains when $Y_2\xi \sim c_2/\omega R \gg 1$,

$$\Delta c_2^{(0)}/c_2 \to (1/2V)\sum_R 4\pi R(c_2/\omega)^2. \tag{34}$$

For $R = 10\mu$m and $\omega = 6.3$MHz, $c_2/\omega R \approx 3$. So we may expect an anomalous behavior in the scattering and sound velocity for a suspension of μm-sized c-drops at $T <30$mK.

Concerning the contribution from $l = 1$ harmonic, the partial scattering amplitude f_1 is given by

$$f_1 = -\frac{R}{3}(q_2R)^2\,\frac{(\rho_2-\rho_1)(2\rho_1+\rho_2)+(\omega R\xi\rho_1\rho_2)^2-3\imath\omega R\xi\rho_1^2\rho_2}{(2\rho_1+\rho_2)^2+(\omega R\xi\rho_1\rho_2)^2}. \qquad (35)$$

Accordingly, we have the elastic and inelastic contributions into the total cross-section $\sigma_{tot}^{(1)} = \sigma_{el}^{(1)} + \sigma_{in}^{(1)}$

$$\begin{aligned}\sigma_{el}^{(1)} &= \frac{4\pi R^2}{3}\left(\frac{\omega R}{c_2}\right)^4\frac{(\rho_2-\rho_1)^2+(\omega R\xi\rho_1\rho_2)^2}{(2\rho_1+\rho_2)^2+(\omega R\xi\rho_1\rho_2)^2}, \\ \sigma_{in}^{(1)} &= 12\pi R^2\left(\frac{\omega R}{c_2}\right)^4\frac{\omega R\xi\rho_1^2\rho_2}{(2\rho_1+\rho_2)^2+(\omega R\xi\rho_1\rho_2)^2}.\end{aligned} \qquad (36)$$

The variation of the sound velocity from $l = 1$ equals

$$\frac{\Delta c_2^{(1)}}{c_2} = \frac{3}{2V}\sum_R\frac{4\pi R^3}{3}\cdot\frac{(\rho_2-\rho_1)(2\rho_1+\rho_2)+(\omega R\xi\rho_1\rho_2)^2}{(2\rho_1+\rho_2)^2+(\omega R\xi\rho_1\rho_2)^2}. \qquad (37)$$

We just note that in the $Y_2\xi \gg 1$ limit the contribution from $l = 1$ harmonic is weaker than from $l = 0$ and remains of the same order of magnitude. As ξ grows, σ and $\Delta c_2/c_2$ increase.

Finally we consider higher harmonics with $l \geq 2$. The partial scattering amplitudes reflect the normal modes of the shape oscillations

$$\begin{aligned}f_l &= -\frac{R}{2l+1}\frac{1}{(2l-1)!!}\left(\frac{\omega R}{c_2}\right)^{2l}\frac{l(\rho_2-\rho_1)-\imath\omega R\xi\rho_1}{(l+1)\rho_1+l\rho_2}\times \\ &\quad\times\frac{\omega^2+\alpha l(l-1)(l+2)/(\rho_2-\rho_1)R^3}{(\omega^2-\omega_0^2)^2-\imath\omega\tau_l(\omega^2-\omega_\infty^2)}\end{aligned} \qquad (38)$$

Here ω_0, ω_∞, and τ_l are defined by Eqs. (17)–(19). The contribution of higher harmonics into σ and $\Delta c_2/c_2$ is obviously much smaller, in spite of its resonance character. However, some resonance behavior as a function of ω may be observable for a homogeneous distribution over the sizes of drops.

4. To conclude, we have calculated the oscillation spectrum of a liquid drop in a phase-separated fluid with a highly mobile interface. It is found that the spherically symmetrical pulsations are strongly softened as compared with the case of two immiscible fluids. Sufficiently small drops can prove to be unstable. The nonspherical shape oscillations can be described as an oscillatory process with some effective relaxation time depending on the growth coefficient. The finite value of the growth coefficient leads both to a frequency shift and to an additional damping of the drop oscillations.

In addition, we have found the cross-section of sound scattering on a drop and the variation of the sound velocity in a suspension of c-drops. We note that the total cross- section and, respectively, sound absorption enhance especially for large growth coefficients $Y_2\xi \gtrsim 1$, as compared with the case of immiscible fluids. The qualitative feature for the finite magnitudes of the growth coefficient is the appearance of the inelastic component of scattering. The sound velocity in a suspension of c-drops grows with an increase of the growth coefficient, displaying also a frequency- and temperature-dependent behavior. Thus, we may expect anomalous behavior of a sound wave propagating across the two-phase system of μm-sized c-drops suspended in a d-phase for the frequencies of 1–10 MHz and larger at temperatures approximately below 30–50 mK due to the high mobility of the interface between the c- and d-phases.

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