

Nuclear spintronics: quantum Hall and nano-systems

Israel D. Vagner

RCQCE -

*Research Center for Quantum Communication Engineering,
at Department of Communication Engineering,
Holon Academic Institute of Technology,
52 Golomb St., Holon 58102, Israel*

and

*Grenoble High Magnetic Fields Laboratory,
Max-Planck-Institute für Festkörperforschung and CNRS,
25 Avenue des Martyrs, BP166, F-38042, Grenoble, Cedex 9, France*

e-mail: vagner_i@hait.ac.il

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Abstract

The electron spin transport in condensed matter, **Spintronics**, is a subject of rapidly growing interest both scientifically and from the point of view of applications to modern and future electronics. In many cases the electron spin transport cannot be described adequately without accounting for the hyperfine interaction between electron and nuclear spins. Under extreme conditions of high magnetic fields, ultra-low temperatures, ultra high isotopical cleanliness etc., the nuclear spins in these systems are very promising candidates for the qubits: the basic elements of emerging quantum memory, logics and hopefully quantum computers.

Here we review the progress in the **Nuclear Spintronics** i.e. in physics and applications of hyperfine interactions in such exotic systems, as superconducting, quantum Hall, mesoscopic and nano-systems.

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1 Introduction

The electron spin transport is playing growing role in the fastly developing new directions of the modern High-Tech electronics, the so-called **Spintronics** [1, 2]. In most solids the conduction electron system is coupled by hyperfine interaction [3, 4] to the underlying system of nuclear spins, which may profoundly modify the electron spin transport at low temperatures, thus leading to **Nuclear Spintronics** effects, which will be described in this review.

On the other hand, the application of high magnetic fields is a very powerful tool for studying the electronic properties of a large variety of metals, semiconductors and superconductors. Due to the Landau quantization of electron motion in sufficiently strong magnetic field, most of the transport properties, such as magnetization, conductivity etc. experience magnetic quantum oscillations (QO) [5]. The Landau quantization is most spectacularly manifest in the electronic magneto-transport in low dimensional conductors. Striking examples are the celebrated quantum Hall effects (QHE) [6].

Apart from the anomalous enhancement of the well known QO in two-dimensional electron systems (2DES) one expects also strong QO in physical properties which are not sensitive to the magnetic field in isotropic three dimensional metals. It was suggested in [7] that in quasi-two-dimensional metals under strong magnetic fields the nuclear spin lattice relaxation rate T_1^{-1} should exhibit strong magnetic oscillations.

This should be compared with the Korringa relaxation law [3] usually observed in three-dimensional normal metals, which results in a magnetic field independent nuclear spin-relaxation rate. This line of research seems to be useful in dense quasi-two-dimensional electronic systems, as is the case of synthetic metals (GIC's etc.) and low-dimensional organic compounds.

A completely new line of research, the hyperfine interaction between nuclear and electron spins in low dimensional and correlated electron systems, has been developed during the two last decades both theoretically [8 - 39], and experimentally [40 - 59]. The growing attention is attracted by the hyperfine interactions in a) QHE: theory [8 - 19] and experiment [40 - 53]; b) mesoscopics and nano-systems: theory [20 - 26] and experiment [54 - 55]; c) normal and superconducting metals: theory [27 - 31] and experiment [58 - 59]. Very recently nuclear spin memory devices [32] and the indirect hyperfine interaction via electrons between nuclear spin qubits in semiconductor based quantum computer proposals [33 - 39] attracts sharply growing attention.

Here I will outline the main theoretical concepts and some experimental achievements in the new and quickly developing field of **Nuclear Spintronics**.

2 Quantized nuclear spin relaxation

2.1 Korringa law

In metals and doped semiconductors, usually, the leading contribution to the spin - lattice relaxation process is due to the *hyperfine Fermi contact* interaction between the nuclear spins and the conduction electron spins [3]. This interaction is represented by the Hamiltonian:

$$\hat{H}_{int} = -\gamma_n \hbar \vec{I}_i \cdot \vec{H}_e, \quad (1)$$

where γ_n is the nuclear gyromagnetic ratio, I_i is the nuclear spin and H_e is the magnetic field on the nuclear site, produced by electron orbital and spin magnetic moments:

$$\vec{H}_e = -g\beta \sum_e \frac{8\pi}{3} \hat{s}_e \delta(\vec{r}_e - \vec{R}_i). \quad (2)$$

Here \vec{r}_e is the electron radius-vector, \hat{s}_e is the electron spin operator, $\beta = e\hbar/m_0c$ is the Bohr magneton and g is the electronic g -factor.

The nuclear spin-lattice relaxation rate T_1^{-1} , caused by the hyperfine Fermi contact interaction between the nuclear spins and the conduction electron spins, is related to the local spin-spin correlation function through the equation:

$$T_1^{-1} = \frac{32\pi^2}{9} \gamma_n^2 g^2 \beta^2 \int_{-\infty}^{\infty} e^{-i\omega_n t} \{ \langle S^+(\mathbf{R}, t) S^-(\mathbf{R}, 0) \rangle \} dt, \quad (3)$$

where $S^+(\mathbf{R})$, $S^-(\mathbf{R})$ are the transverse components of the electron spin density operator at the nuclear position \mathbf{R} , and ω_n is the nuclear magnetic resonance frequency.

The rate of the nuclear spin-relaxation in metals is, usually, proportional to the temperature and to the square of the electronic density of states at the Fermi energy (the Korringa law [3]). This follows from the following expression:

$$\frac{1}{T_1} \propto \int_0^{\infty} |\langle i | V | f \rangle|^2 \rho(E_i) \rho(E_f) f(E_i) [1 - f(E_f)] \delta(E_f - E_i + \gamma_N H_0). \quad (4)$$

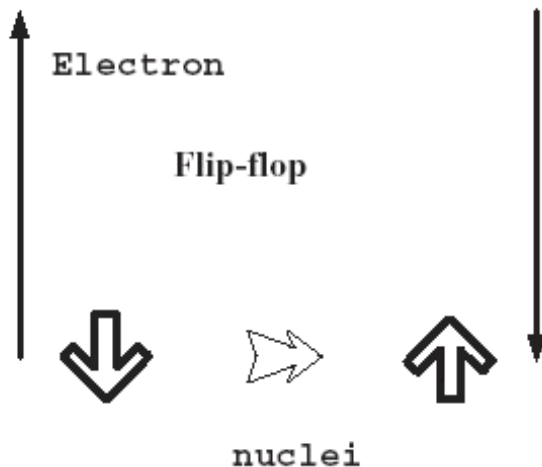


Figure 1: Simultaneous electron and nuclear spin flips induced by the contact hyperfine interaction.

At low temperatures: $f(E)[1 - f(E)] \propto k_B T \partial f / \partial E$, where $k_B T$ is the temperature in the energy units, we arrive at the linear in temperature dependence of $1/T_1$

$$T_1^{-1} \sim k_B T \rho^2(E_F), \quad (5)$$

which is the well known Korringa law [3]. Here $\rho(E_F)$ is the electron density of states at the Fermi level.

2.2 Activation law for $1/T_1$ in QHE systems.

In high magnetic fields and in systems with reduced dimensionality this simple argumentation does not hold, since the electron spectrum acquires field induced [8] or size quantized [20, 24] energy gaps.

It was conjectured in [8] and confirmed experimentally, see e.g. [41, 44], that in QHE systems the nuclear spin relaxation rate should have an activation behavior

$$T_1^{-1} \sim \exp \left\{ -\frac{\Delta(B)}{k_B T} \right\}, \quad (6)$$

where $\Delta(B)$ is either $g\mu_B B$, the electron Zeeman gap (odd filling factors) or $\hbar\omega_c$, the Landau levels gap (even filling factors), instead of the usual

Korringa law. The discreteness of the electron spectrum manifests, at finite temperatures, in an activation type of the magnetic field dependence of the nuclear spin relaxation rate, T_1^{-1} as it seen from Eq. (6). This behaviour is similar to that of the magnetoresistance ρ_{xx} in the QHE, [8], see Fig. 2.

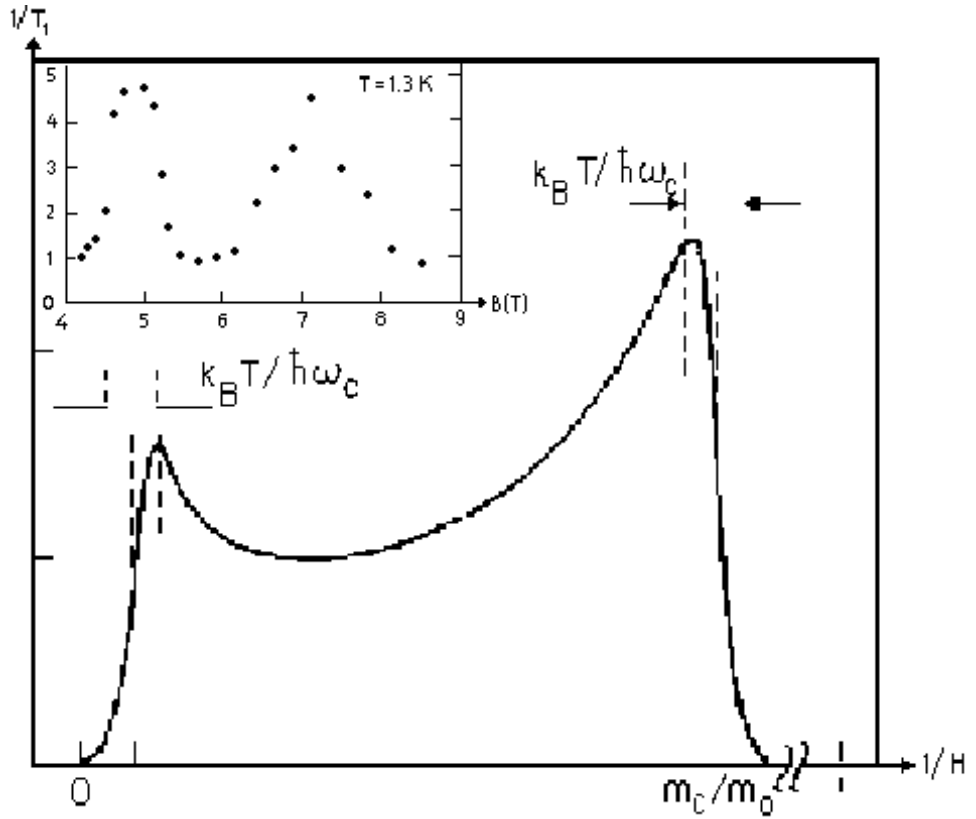


Figure 2: Theoretically predicted [8], magnetic field dependence of the nuclear spin relaxation rate in quantum Hall systems. The inset shows schematically the experimental data [41]. In the inset the vertical axis is in msec^{-1} .

2.2.1 Energy conservation

This unusual magnetic field dependence of the nuclear spin relaxation reflects the fact that the energy gaps in the spectrum of two-dimensional electrons in strong magnetic fields (either Zeeman splitting or the Landau levels

gap) are three orders of magnitude larger than the nuclear Zeeman energy. Indeed, the energy needed to reverse the spin of an electron in the external magnetic field H_0 is $\Delta E_{el} = 2g\mu_B H_0$, which is much larger (by a factor of $M_n/m_e \simeq 10^3$, M_n and m_e being the nuclear and free electron masses) than the energy $\gamma_n H_0$ provided by reversing the nuclear spin. Therefore the delta function in Eq. (2) can not be realized and the simultaneous spin flip of the nuclear and the electron spins (flip-flop) in Landau levels, Fig. 3, is severely restricted by the energy conservation.

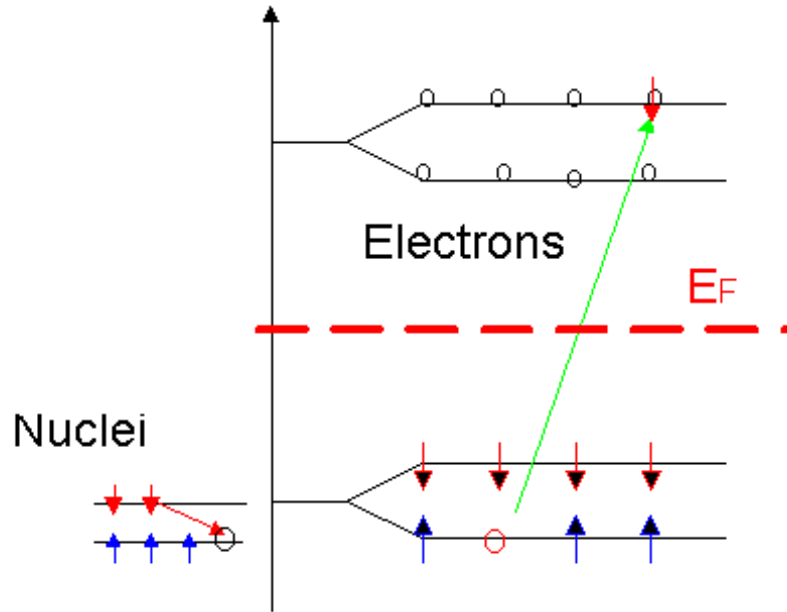


Figure 3: Flip-flop in Landau levels. Even filling.

On the other hand, in isotropic $3D$ electron systems, in a strong magnetic field : $\hbar\omega_c > k_B T$, the kinetic energy of the electron motion parallel to the field should change in order to ensure the energy conservation of the process shown in Fig. 4. Thus, in the "isotropic" $3D$ model, the electron spin - flip will be accompanied by a simultaneous change of the Landau level and of the kinetic energy parallel to the field, E_z according to

$$\Delta\epsilon_z = \hbar\omega_c(n' - n) + \gamma_n H_0 - \hbar\omega_z. \quad (7)$$

While this is impossible for an ideal $2D$ system in a strong magnetic field, it may take place in quasi-two-dimensional conductors, as is the case

in superlattices, for certain regions of parameters.

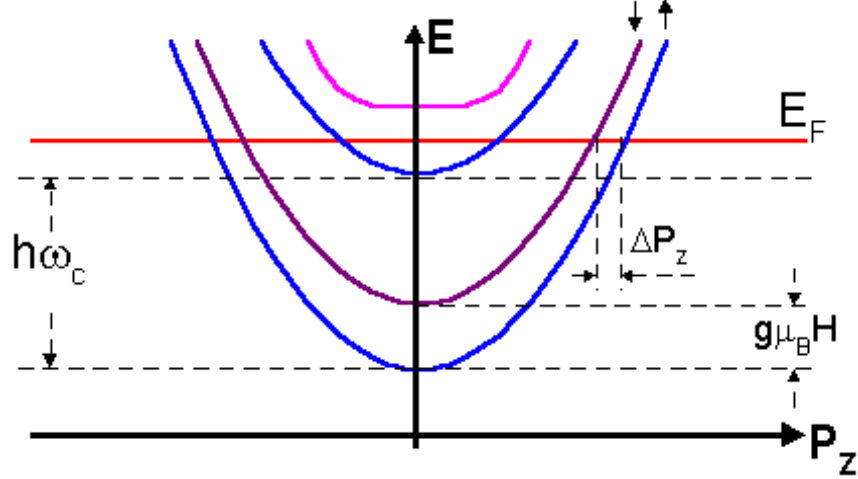


Figure 4: Spin splitted Landau levels in 3D electron systems.

Because of the existence of energy gaps in the electron spectrum of a 2DES under strong magnetic fields (the QHE systems), finite nuclear spin relaxation times T_1 could be expected only if 2DES is subjected to different kinds of external potentials.

2.2.2 Short range impurities

Broadened Landau levels may overlap, thus providing a nonzero density of states of both spin projections at the Fermi energy. This will result in finite relaxation time for excited nuclear spins interacting with the conduction electrons on the Landau levels. Description of these processes, performed in the framework of the finite temperature Matsubara diagram techniques, is presented in [9, 10].

2.2.3 Edge states

An important question in the quantum Hall effect theory is the role of the edge states: electronic orbits magnetically confined to the sample boundary. While the number of edge states may be small compared to the "bulk" states, their contribution to T_1 can be very important, since they possess a homogeneous energy spectrum [12].

In sufficiently clean heterojunctions, however, where the fractional QHE (FQHE) and Wigner crystallization could be observed, the mechanisms mentioned above are extremely inefficient.

2.2.4 Phonon assisted mechanism

At finite temperatures the energy conservation in the flip-flop process can be fulfilled by absorbing a phonon [13-15].

The mechanism, considered in [13], consists of two processes in which, first, electron and nuclear spins are simultaneously reversed by the hyperfine interaction, and, second, only electron spins are reversed by their coupling to the lattice strain. In these processes, a phonon is either absorbed or emitted to satisfy the energy conservation law.

Both the Zeeman split-level energy ($\hbar\omega_z$) and chemical potential (μ) depend on a magnetic field. The dominant contribution to nuclear relaxation comes from the transition between two spin-split levels that are near the chemical potential. The contributions from other Landau levels that are far away from the chemical potential are proportional to $\exp[-n\hbar\omega_c/k_B T]$ and are negligible under the QHE condition. The value at a minimum depend strongly on the value of exchange interaction.

At low temperatures ($k_B T \ll \hbar\omega_z$), the nuclear spin relaxation rate varies exponentially with inverse temperature (i.e., $T_1^{-1} \propto \exp[-\hbar\omega_z/k_B T]$) because the number of thermally excited phonons in the reservoir becomes exponentially small as the temperature is decreased. At higher temperatures ($\hbar\omega_z \ll k_B T \ll \hbar\omega_c$), on the other hand, the relaxation rate varies linearly with temperature (i.e., $T_1^{-1} \propto k_B T/\hbar\omega_z$).

Nucleus-mediated spin-flip transitions in GaAs quantum dots have been studied in [14]. In these papers the electron spin relaxation times were calculated taking into account the mechanism, where the phonons provide the necessary energy and nuclear spins take care for the spin conservation during the electron transition between Zeeman split states in quantum dots.

2.2.5 Dipole - dipole interaction

The nuclear spin relaxation in 2DEG, associated with the dipole-dipole interaction between the nuclear and the electronic spins was studied by Ovchinnikov et al. [16]. The principle physical difference between these two mechanisms is that while in the contact interaction the total spin should be conserved and an electron has to flip its spin in order to relax the nuclear spin, the conserved quantity in the dipole-dipole interaction is the total

(spin plus orbital) momentum. Therefore an electron can relax a nuclear spin without changing its own spin state, just by shifting its center of orbit. This mechanism should be prevailing in clean samples, i.e. under the FQHE conditions.

3 Electron interaction

3.1 Spin-excitons

Much of the recent attention paid to hyperfine interactions under conditions of the quantum Hall effect is connected with correlation effects in 2DES. This is based on the notion of a spin-exciton: the elementary excitation over the Zeeman gap dressed by the Coulomb interaction [60, 61]. This results in a strong enhancement (up to a factor of 100, as is the case in GaAs) of the effective $g(k)$ -factor.

Due to the Coulomb interaction the spin-excitons are bound states of electron-hole pairs which, unlike the individual electrons or holes, can propagate freely under the influence of a magnetic field due to their zero electric charge. These elementary excitations are, therefore, chargeless particles with a nearly parabolic dispersion in the low- k limit. At $k = 0$ the gap is equal to the "bare" Zeeman splitting.

The energy spectrum of spin-excitons on the ground Landau level: $n=0$ is [60, 61]:

$$E_{ex}^{sp}(k) = |g| \mu_B H_0 + \sqrt{\frac{\pi}{2}} [1 - I_0(\frac{k^2}{4}) \exp(-\frac{k^2}{4})] \frac{e^2}{\kappa a_H}. \quad (8)$$

In the parabolic approximation (small exciton momenta), the dispersion relation reads:

$$E_{ex}^{sp}(k) \approx |g| \mu H_0 + \frac{k^2}{2m_{se}}, \quad (9)$$

where

$$\frac{1}{2m_{se}} \equiv \frac{1}{4} \sqrt{\frac{\pi}{2}} \frac{e^2 a_H}{\chi \hbar^2}$$

is the definition of the spin-exciton mass.

The invariance of the energy gap with respect to the electron-electron interaction is associated with the fact that in creating a quasielectron-quasihole pair excitation at the same point in space (i.e. with center of

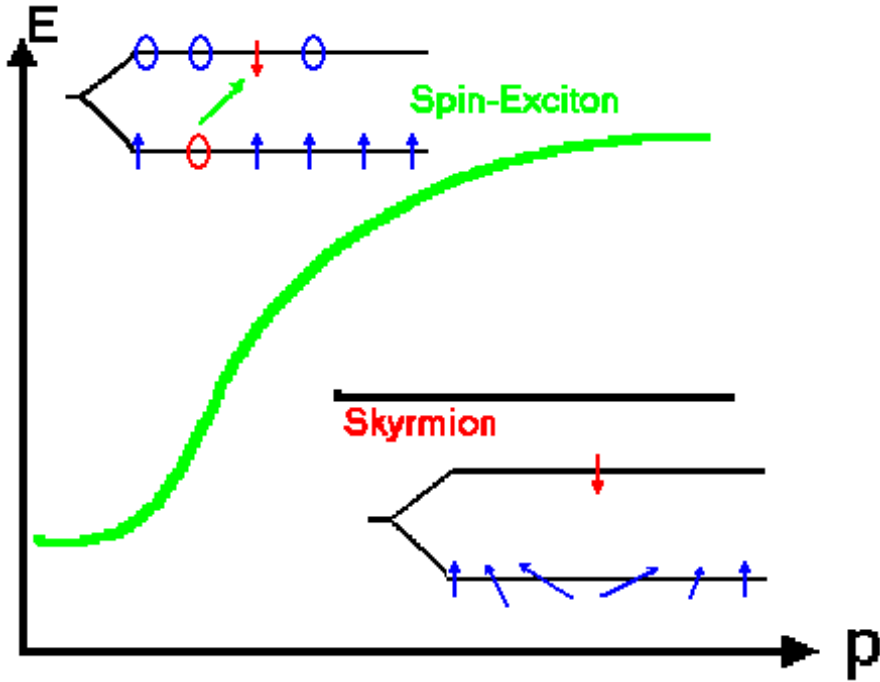


Figure 5: 2D Spin exciton and skyrmion dispersion: the energy gap of large momentum excitons is two times larger than the skyrmion's one. In the insets the 2D electron spin arrangements for spin excitons (left) and skyrmions (right) is shown.

mass momentum $k = 0$) the energy decrease due to the Coulomb attraction is exactly cancelled by the increase in the exchange energy. Thus the energy gap for the creation of a widely separated (i.e. with $k \rightarrow \infty$) quasielectron-quasihole pair (large spin-exciton) is equal to the exchange energy associated with the hole.

External potentials, like long range potential fluctuations, periodically modulated heterostructures etc., may reduce substantially the spin-exciton gap.

3.1.1 Periodically modulated 2DES

In [62] the spin-exciton in a periodically modulated two-dimensional electron gas under strong magnetic fields was investigated. It was shown there that a periodic external potential, with a period in the submicron range, can

remove the energy gap in the spin-exciton dispersion at certain values of the spin-exciton momenta. This may result in several interesting phenomena. For example, the rate of nuclear spin relaxation and depolarization processes in heterojunctions will be strongly enhanced.

It is easy to see why the Zeeman energy gap can be removed by the modulation potential in an ideal model of noninteracting electrons: consider a noninteracting electron-hole pair with opposite spins, which are subject to an external magnetic field and a periodic potential with amplitude V_0 and period a . By appropriately selecting the spatial separation between the electron and the hole along the modulation direction the energy difference between them can be reduced to zero. The minimal amplitude V_0 required to satisfy this condition is $\varepsilon_{sp}/2$, the spatial separation between the electron and the hole involved is half of the modulation period a , and the corresponding wave number (along the y-direction) is $k_y = a/2l_B^2$. The picture is significantly more complicated, however, in the presence of the strong Coulomb interaction, where bound spin-excitons exist.

3.2 Quantum vacuum fluctuations

Maniv et al. [19], have considered the effect of vacuum quantum fluctuations in the QH ferromagnetic state on the decoherence of nuclear spins. It was shown there that the virtual excitations of spin excitons, which have a large energy gap (on the scale of the nuclear Zeeman energy) above the ferromagnetic ground state energy, lead to fast incomplete decoherence in the nuclear spin system. It is found that a system of many nuclear spins, coupled to the electronic spins in the 2D electron gas through the Fermi contact hyperfine interaction, partially loses its phase coherence during the short (electronic) time \hbar/ε_{sp} , even under the ideal conditions of the QHE, where both T_1 , and T_2 are infinitely long. The effect arises as a result of vacuum quantum fluctuations associated with virtual excitations of spin waves (or spin excitons) by the nuclear spins. The incompleteness of the resulting decoherence is due to the large energy gap of these excitations whereas the extreme weakness of the hyperfine interaction with the 2D electron gas guarantees that the loss of coherence of a single nuclear spin is extremely small.

The manipulation of the nuclear spins is carried out through spin flip-flop processes, associated with the 'transverse' part of the interaction Hamiltonian \hat{H}_{en} , i.e. $A \sum_j \left[\hat{I}_{j,+} \hat{S}_-(\mathbf{r}_j) + \hat{I}_{j,-} \hat{S}_+(\mathbf{r}_j) \right]$, where

$$\hat{I}_{j,+} = \hat{I}_{j,x} + i\hat{I}_{j,y}, \hat{I}_{j,-} = \hat{I}_{j,x} - i\hat{I}_{j,y}$$

are the transverse components of the nuclear spin operators, and

$$\widehat{S}_+(\mathbf{r}) = \widehat{\psi}_\downarrow^\dagger(\mathbf{r})\widehat{\psi}_\uparrow(\mathbf{r}), \widehat{S}_-(\mathbf{r}) = \widehat{\psi}_\uparrow^\dagger(\mathbf{r})\widehat{\psi}_\downarrow(\mathbf{r})$$

are the corresponding components of the electron spin density operators. Here $\widehat{\psi}_\sigma(\mathbf{r})$, $\widehat{\psi}_\sigma^\dagger(\mathbf{r})$ are the electron field operators with spin projections $\sigma = \uparrow, \downarrow$. The 'longitudinal' part of \widehat{H}_{en} , $A \sum_j \widehat{I}_{j,z} \widehat{S}_z(\mathbf{r}_j)$, which commutes with the Hamiltonian \widehat{H}_0 , and so leaves the nuclear spin projections along \mathbf{B}_0 unchanged, can still erode quantum coherence in the nuclear spin system [63].

To simplify the analysis it is assumed that the nuclei under study have spin 1/2, and so the corresponding spin operators are expressed in terms of Fermionic creation and annihilation operators, $\widehat{c}_{j,\sigma}$, as

$$\widehat{I}_{j,+} = \widehat{c}_{j,\uparrow}^\dagger \widehat{c}_{j,\downarrow}, \widehat{I}_{j,-} = \widehat{c}_{j,\downarrow}^\dagger \widehat{c}_{j,\uparrow}, \text{ and } \widehat{I}_{j,z} = \frac{1}{2} \left(\widehat{c}_{j,\uparrow}^\dagger \widehat{c}_{j,\uparrow} - \widehat{c}_{j,\downarrow}^\dagger \widehat{c}_{j,\downarrow} \right).$$

In this case the transverse components $\widehat{I}_{j,+}$, $\widehat{I}_{j,-}$, are up to a proportionality constant, just the off diagonal elements (or coherences) of the density matrix of a single nuclear spin (qubit) [64, 3].

The decay of these elements with time, which determines the rate of decoherence of a single qubit, can be thus found from the equations of motion for the operators $\widehat{c}_{j,\sigma}$ in the Heisenberg representation

$$\widehat{c}_{j,\sigma}(t) = e^{i\widehat{H}t/\hbar} \widehat{c}_{j,\sigma} e^{-i\widehat{H}t/\hbar}.$$

Considering a single nuclear spin and evaluating its rate of decoherence due to the coupling with a 'bath' of spin excitons, assuming that initially, at time $t = 0$, the electronic system is in its ground (QH ferromagnetic) state $|0\rangle$, and neglecting the effect of the nuclear spins on the electronic (bath) states, and averaging over the 'bath' states, one finds to lowest order in the hyperfine interaction parameter α that the time dependence of the coherence I_+ is given by $I_+(t) = I_+(0) J(t)$, where $J(t) = \exp[i\Omega(t) - \Gamma(t)]$. This result is similar to the expression found by Palma et al. [63] in an artificial model of pure decoherence, i.e. when energy transfer between the qubit and its environment is not allowed.

The remarkable feature of this expression is due to the presence of the energy gap ε_{sp} in the spin exciton spectrum, which is typically much larger than the nuclear Zeeman energy $\hbar\omega_n$. During a short time scale, of the order of \hbar/ε_{sp} , the coherence $I_+(t)$ of a single nuclear spin diminishes and

then saturates for a very long time (i.e. of the order of the relaxation time T_2) at $I_+(0) e^{-\eta\tilde{C}^2}$, where

$$\eta = \int_0^\infty \frac{\tilde{k}d\tilde{k}e^{-\frac{1}{2}\tilde{k}^2}}{[\tilde{E}_{ex}(k)]^2} \sim 1.$$

For $GaAs/Al_xGa_{1-x}As$ heterostructure the coupling constant \tilde{C} is typically of the order of 10^{-4} [17].

3.3 Long-range random potential

3.3.1 Nuclear spin relaxation rate T_1

Iordanskii et al. [11] have studied nuclear spin relaxation taking into account the creation of spin-excitons [60] in the flip-flop process. The energy for the creation of a spin-exciton can be provided by the long range impurity potential in a process, where the electron turns its spin while its center of orbit is displaced to a region with lower potential energy.

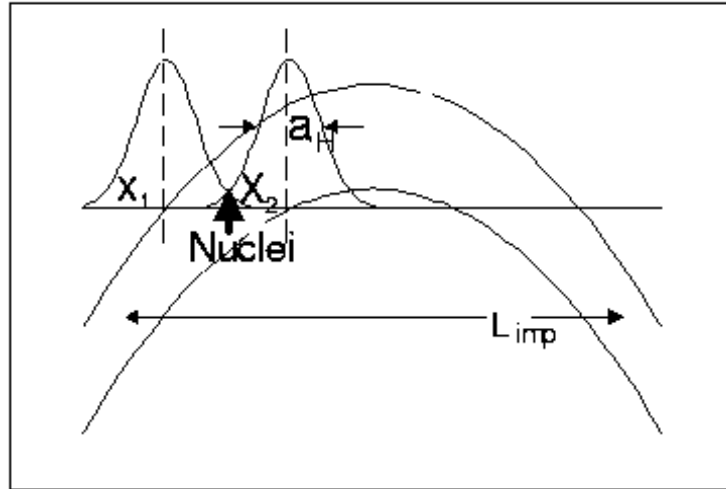


Figure 6: Long-range electrostatic potential, created by a remote impurity, provides the energy to reverse the electron spin in the nuclear-electron flip-flop process.

As shown in Fig. 6, the overlap of the initial and final location of the electron wave functions, centered at x_1 and x_2 respectively, is: $exp[-(x_1 -$

$x_0)^2/a_H - (x_2 - x_0)^2/a_H^2]$. Here x_0 is the nuclear position. Nuclear spin relaxation by the conduction electron spin in the vicinity of a potential fluctuation is effective when the nuclear spin is positioned in the region of the overlapping initial and final states of the electron wave function.

The energy conservation in the spin-exciton creation process can be written in the form:

$$\mu_B g H_0 + E(p) = x \nabla U.$$

This expression defines the gradient of electric potential caused by the impurity, sufficient to create a spin-exciton during a flip-flop process. The probability of finding such a fluctuation is exponentially small: $\exp[-(\nabla U)^2/2 < \nabla U^2 >]$.

The momentum \mathbf{p} of the spin-exciton is small and therefore the expansion in \mathbf{p} can be performed everywhere in the final expression for electronic density of states (DOS):

$$\text{DOS} = \int \text{Im}G(E, x, x) dx = e^{-S}. \quad (10)$$

Here S depends on the following combination of physical parameters:

$$S \propto |g| \frac{(\mu H E_c) L^2}{R''(0) a_H^2}, \quad (11)$$

$R''(r) \equiv d^2 R(r)/dr^2$ reaches the maximum, usually, at $r = 0$.

A complete expression for the nuclear depolarization rate from the golden rule formula at $T=0$ [11] is given by

$$T_1^{-1} \propto \nu_{\uparrow} (1 - \nu_{\downarrow}) \frac{1}{2\pi^2} \int \text{Im}G(k, \omega) \delta(\omega - \omega_N) L_n(k^2) e^{-\frac{k^2}{2}} d^2 k d\omega,$$

where $\nu_{\uparrow}, \nu_{\downarrow}$ are filling factors for the electrons with spin up and spin down, respectively, n is the highest occupied Landau level and $L_n(k^2/2)$ are the Laguerre polynomials. Here the system of units is used, where $a_H^2 = c\hbar/eH = 1$ and $\hbar = 1$.

3.3.2 Decoherence due to impurities, T_2

The dynamics of the nuclear spins is governed by their interactions with each other and with their environment. In the regime of interest, these interactions are mediated by 2DES. Various time scales are associated with this dynamics. The relaxation time T_1 is related to energy exchange and

thermalization of spins. Quantum mechanical decoherence/dephasing will occur on the time scale T_2 .

Theoretical calculation of the nuclear-spin dephasing/decoherence time scale T_2 for in QHE systems with impurities was presented in [18]. The hyperfine interaction between the electron and the nuclear spins, H_{ne} , can be split into two parts

$$H_{ne} = H_{\text{diag}} + H_{\text{offdiag}}, \quad (12)$$

where H_{diag} corresponds to the coupling of the electrons to the diagonal part of nuclear spin operator \mathbf{I}_n , and H_{offdiag} — to its off-diagonal part. It follows [18], that H_{diag} can be incorporated into the nuclear-spin energy splitting, redefining the Hamiltonian of the nuclear spin as $H_n = \frac{1}{2}\Gamma\sigma_z$, where $\Gamma = \gamma_n(B + B_{\text{Knight}})$.

The relevant terms in the full Hamiltonian can be expressed solely in terms of the nuclear-spin operators and spin-excitation operators [18]. It is assumed there that initially, at time $t = 0$, the nuclear spin is polarized, while the excitons are in the ground state, $|\Psi(0)\rangle = |-\rangle \otimes |\mathbf{0}\rangle$ where $|-\rangle$ is the polarized-down (excited) state of the nuclear spin and $|\mathbf{0}\rangle$ is the ground state of spin-excitons. Since the Hamiltonian conserves the total number of elementary excitations in the system, the most general wave function can be written as

$$|\Psi(t)\rangle = \alpha(t) |-\rangle \otimes |\mathbf{0}\rangle + \sum_{\mathbf{k}} \beta_{\mathbf{k}}(t) |+\rangle \otimes |\mathbf{1}_{\mathbf{k}}\rangle \quad (13)$$

with $|+\rangle$ corresponding to the nuclear spin in the ground state and $|\mathbf{1}_{\mathbf{k}}\rangle$ describing the single-exciton state with the wave vector \mathbf{k} .

Equations of motion for the coefficients α and $\beta_{\mathbf{k}}$, derived from the Schrödinger equation are:

$$i\hbar\dot{\alpha} = \frac{1}{2}\Gamma\alpha + \sum_{\mathbf{k}} g_{\mathbf{k}}\beta_{\mathbf{k}} \text{ and } i\hbar\dot{\beta}_{\mathbf{k}} = -\frac{1}{2}\Gamma\beta_{\mathbf{k}} + E_{\mathbf{k}}\beta_{\mathbf{k}} + \sum_{\mathbf{q}} \phi_{\mathbf{k},\mathbf{q}}\beta_{\mathbf{q}} + g_{\mathbf{k}}\alpha.$$

The relaxation rate and the added phase shift of the nuclear-spin excited-state probability amplitude $\alpha(t)$ are given by the real and imaginary parts of the pole, respectively:

$$\frac{1}{T_1} = \frac{\pi}{\hbar} \sum_{\mathbf{k}} g_{\mathbf{k}}^2 \delta(\Gamma - E_{\mathbf{k}})$$

and

$$\Delta\omega = P \sum_{\mathbf{k}} \frac{g_{\mathbf{k}}^2}{\Gamma - E_{\mathbf{k}}}$$

so that $\alpha(t) \propto \exp[-t/T_1 + i\Delta\omega t]$. It is obvious that due to the large gap in the spin-exciton spectrum, $\Gamma \ll \Delta$, the energy conservation in the flip-flop process can not be satisfied, and so in the absence of interaction with impurities, $T_1 = T_2 = \infty$.

Interactions with impurities will modify these solution, and, as a consequence, the energy conservation condition. In particular, if the impurity potential is strong enough, it can provide additional energy to spin-excitons, so that their energy can fluctuate on the scale of order Γ thus making nuclear-spin relaxation possible. This mechanism was identified in [11, 10], and it corresponds to large fluctuations of the impurity potential $U(\mathbf{r})$, which usually occur with a rather small probability, so T_1 is very large for such systems.

The perturbative solution does not describe the energy relaxation (T_1), but it does yield the phase shift due to the impurity potential. This phase shift, when averaged over configurations of the impurity potential, produces a finite dephasing time, T_2 , which can be calculated considering the reduced density matrix of the nuclear spin, given by

$$\rho_n(t) = [\text{Tr}_e |\Psi(t)\rangle \langle \Psi(t)|]_U. \quad (14)$$

Here the trace is partial, taken over the states of the spin-excitons, while the outer brackets denote averaging over the impurity potential. The trace over the spin-excitons can be carried out straightforwardly because within the leading-order perturbative approximation used here they remain in the ground state; all excitations are virtual and contribute only to the phase shift. The diagonal elements of $\rho_n(t)$ are not influenced by virtual excitations and remain constant.

The off-diagonal elements of $\rho_n(t)$ contain the factors $\exp[\pm i\Delta\omega_U t]$. It is the averaging of these quantities over the white-noise impurity potential $U(\mathbf{r})$ that yields dephasing of the nuclear spin. This averaging can be done by utilizing the relation $[\exp(i\phi)]_U = \exp[-\frac{1}{2}([\phi]_U)^2]$.

3.4 Nuclear spin diffusion

Apart from the direct nuclear spin relaxation, important information about the electron system can be obtained from nuclear spin diffusion processes.

This is the case when the nuclear spins are polarized in a small part of a sample as it was experimentally observed in [42, 54].

To explain these experimental observations, Bychkov et al. [17] have suggested a new mechanism for indirect nuclear spin coupling via the exchange of spin excitons. The spin diffusion rate from a given nuclear site \vec{R}_a within the polarized region is proportional to the rate of transition probability $P(\vec{R}_a)$ for the polarization of the nuclear spin \downarrow , located at \vec{R}_a , to be transferred to a nuclear spin \uparrow , positioned at \vec{R}_b , outside the polarized region, via the exchange of virtual spin excitons, Fig. 7. The virtual character of the spin-excitons, transferring the nuclear spin polarization, removes the problem of the energy conservation, typical for a single flip-flop process. Furthermore, the virtual spin-excitons are neutral entities, which can propagate freely in the presence of a magnetic field. In this model the electron interactions play a crucial role: the kinetic energy of a spin exciton is due to the Coulomb attraction between the electron and the hole. Thus the proposed mechanism yields the possibility of transferring nuclear spin polarization over a distance much longer than the magnetic length ℓ_B . The long range nature of this mechanism is of considerable importance when the size of the region of excited nuclear spins, L_{ex} , is much larger than the magnetic length ℓ_B .

As it is shown in [17], the potential of the nuclear spin - spin interaction, mediated by the exchange of spin-excitons, is a monotonic function of the distance between the two nuclei with the asymptotics:

$$U(R_{ab}) \propto -\sqrt{\frac{d}{R_{ab}}} e^{-\frac{R_{ab}}{d}},$$

where

$$d \equiv \frac{\ell_B}{2} \sqrt{\frac{\epsilon_c}{\epsilon_{sp}}}.$$

This is typical for the interaction, mediated by the exchange of quasiparticles with an energy gap at $q \rightarrow 0$, as is the case for the spin-exciton dispersion. The range ΔR of this potential is determined by the critical wave number

$$k_0 = \frac{2}{\ell_B} \sqrt{\frac{\epsilon_{sp}}{\epsilon_c}}$$

as it follows from the uncertainty principle: $\Delta R \cdot k_0 \simeq 1$. The negative sign of this interaction corresponds to attraction between the nuclear spins and may cause, at sufficiently low temperatures, a ferromagnetically ordered nuclear state in QHE systems.

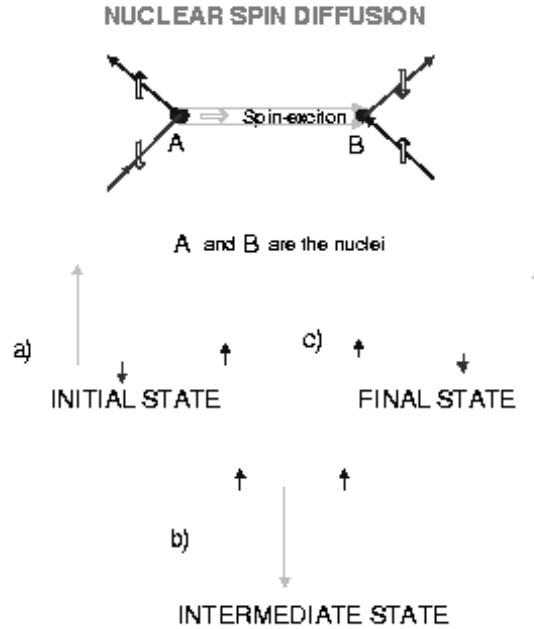


Figure 7: Indirect interaction between two nuclear spins via conduction electron spin.

3.5 Skyrmions and skyr-nuons

Spin-excitons constitute the building blocks, from which more complex spin textures can be formed. For example, at finite densities the spin excitons "condense" [65] into skyrmion-antiskyrmion pairs. These unusual topological excitations [66, 67] in the spin distribution of real 2DES were observed, in NMR experiments, near filling factor $\nu = 1$ [43, 44]. Further evidences for skyrmions in 2DES were found in transport and optical experiments.

Skyrmions, in QHE systems, are the topologically nontrivial spin excitations around filling factor $\nu = 1$ [66] which arise as a condensate of interacting spin excitons [65]. The Coulomb interaction acts to enlarge the Skyrmion size while the Zeeman splitting tends to collapse Skyrmions. The interplay between these factors determines the final distribution of spins within a Skyrmion, and its characteristic length scales. The resulting radius R corresponds to the region where both these energies are of the same value,

and grows weakly to infinity as the g-factor goes to zero [68], thus reflecting the importance of the long range Coulomb repulsion associated with the Skyrmion charge in the zero g-factor limit.

A canonical $u - v$ transformation from the fully polarized ground state, where all spins are oriented along a single axis, to a state, consisting of a macroscopic number of differently oriented spins, each of which is slightly rotated with respect to its nearest neighbors in space was developed in [65]. It is found also that contrary to the lowest Landau level, these energies are positive for both skyrmions and antiskyrmions.

It should be noted that the spin-rotation transformation employed in [65] is unitary and does not change the total number of electrons. Thus by going to the new state $|\psi\rangle$ from the fully polarized ground state $|\psi_0\rangle$ the total topological charge does not change. This can be done if the topological defects are created in pairs of widely separated skyrmions and the corresponding antiskyrmions with equal and opposite charges. The total energy of such a skyrmion-antiskyrmion pair, with winding number $Q = 1$, is exactly equal to half the total energy required to create a well separated electron-hole pair (large spin exciton).

If the Zeeman splitting is not neglected, however, the increase in magnetic energy associated with the rotated spins determines a length scale for the spatial size of the defect. To take into account Zeeman spin splitting the Hartree-Fock (HF) energy functional should be corrected by the additive term $-\frac{\epsilon_{sp}}{4\pi} \int d^2r (\hat{z} \cdot \vec{n})$ [68], where ϵ_{sp} is the Zeeman splitting energy. The corresponding equation for the vectorial field $\vec{n}(\vec{r})$ can be obtained by variation of the corrected energy functional with respect to \vec{n} under the constraint $|\vec{n}|^2 = 1$. The result of such a calculation is:

$$\Delta \vec{n} - \vec{n}(\vec{n} \cdot \Delta \vec{n}) = l_{sk}^{-2} [(\hat{z} \cdot \vec{n})\vec{n} - \hat{z}], \quad (15)$$

where

$$l_{sk}^{-2} \equiv 4\epsilon_{sp}/E(0)l_H^2 = (4/\sqrt{2\pi})|g|(\tilde{a}_B/l_H^3)$$

and

$$\tilde{a}_B \equiv \frac{\kappa \hbar^2}{m_0 e^2}$$

is the effective Bohr radius (note that m_0 is the free electron mass).

In NMR experiments on skyrmions [43,46-48,51] the nuclear spins are strongly polarized. The sample inhomogeneity may result in a strong inhomogeneity of the hyperfine field, Fig. 8, and therefore spatially varying electron Zeeman splitting.

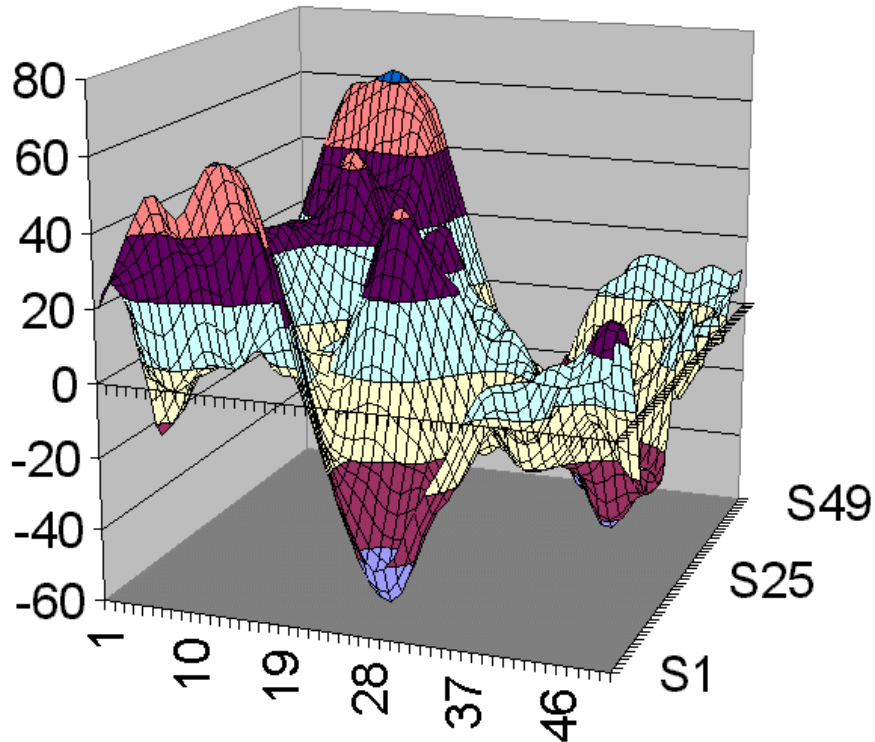


Figure 8: Random hyperfine field potential.

This may result in a strong localization of skyrmions [69, 51], resulting in the combined topologically nontrivial electron-nuclear spin excitation, the skyr-nuon, Fig. 9.

4 Nonequilibrium nuclear spin polarisation

As we have seen in previous sections, the nuclear spin polarization, once created, remains finite for macroscopically long times [4]. Intensive experimental studies [41, 54, 44] of this phenomenon have provided a more detailed knowledge on the hyperfine interaction between the nuclear and electron spins in heterojunctions and quantum wells. It was observed that the nuclear spin relaxation time is rather long (up to 10^3 sec) and the hyperfine field acting on the charge carriers spins is extremely high, up to $10^4 G$ [41, 54]. The nontrivial physics in this subject is based on a fact that the

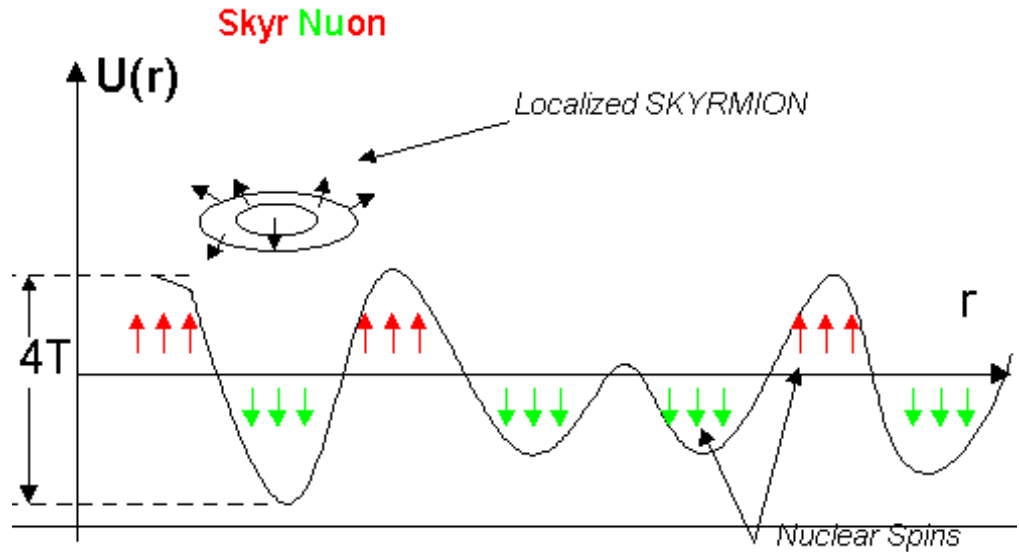


Figure 9: Skyr-nuons, i.e. finite size skyrmions, localised by a random hyperfine field.

discrete nature of the electron spectrum in these systems will result in exponentially long dependence of the nuclear spin relaxation times T_1 on the system parameters [8], i.e. $T_1 \sim \exp\{\Delta/T\}$ (here Δ is the electron energy level spacing and T is the temperature). We assume here that similar law should take place also in the nanostructures with well defined size quantization of the electron spectrum. Note that in this case T_1 is very sensitive to the potential fluctuations, caused by the inhomogeneous distribution of impurities in a heterojunction [11].

4.1 Hyperfine Aharonov-Bohm effect

A family of new physical effects in nanostructures with strong spin-orbital coupling appears when the electron spin degeneracy is lifted by a hyperfine field of polarized nuclei. Indeed, the combined action of a strong nuclear polarization and the spin-orbit interaction, breaks the time reversal symmetry in a mesoscopic system. A good illustration for such **meso-nucleo-spinic** effects is the hyperfine Aharonov-Bohm effect (HABE) in mesoscopic rings [20], caused by hyperfine interaction, as is explained in what follows.

Persistent currents (PC) in mesoscopic rings reflect the broken clock

wise-anticlock wise symmetry of charge carriers momenta caused, usually, by the external vector potential. Experimentally PCs are observed when an adiabatically slow time dependent external magnetic field is applied along the ring axis. The magnetic field variation results in the oscillatory, with the magnetic flux quantum $\Phi_0 = \frac{hc}{e}$ (or its harmonics) period behavior of the diamagnetic moment (the PC), which is the manifestation of the usual Aharonov-Bohm effect (ABE).

It was suggested in [20] that in a quantum ring with a nonequilibrium nuclear spin population the persistent current will exist, even in the absence of external magnetic field. It is shown there, that the ABE like oscillations of PC with time will appear, during the time interval of the order of nuclear spin relaxation time T_1 . The physics of this phenomenon can be understood along the following lines.

The hyperfine field, caused by the nonequilibrium nuclear spin population breaks the spin symmetry of charged carriers. Combined with a strong spin - orbital (SO) coupling, in systems without center of inversion [70], it results in the breaking of the rotational symmetry of diamagnetic currents in a ring. Under the topologically nontrivial spatial nuclear spin distribution, the hyperfine field produces an adiabatically slow time variation of the Berry phase of the electron wave function.

The time variation of this topological phase results in observable oscillations of a diamagnetic moment (the PC). It is one of a series of "meso-nucleo-spinic" effects, which may take place in mesoscopic systems with broken symmetry, due to the combined action of the hyperfine field and spin-orbital interaction.

Due to the contact hyperfine interaction Eq. (1) it follows that once the nuclear spins are polarized, i.e. if $\langle \sum_i \mathbf{I}_i \rangle \neq 0$, the charge carriers spins feel the effective, hyperfine field $B_{hyperf} = B_{hyperf}^0 \exp(-t/T_1)$ which lifts the spin degeneracy even in the absence of external magnetic field. In GaAs/AlGaAs one may achieve the spin splitting due to hyperfine field of the order of the one tenth of the Fermi energy [41, 54].

Let us suppose therefore that the charge carriers spin orientation is partially polarized during the time interval of the order of T_1 . It is quite obvious that the topologically nontrivial spin texture combined with the spin-orbit interaction will result in a PC.

Taking the Bychkov-Rashba term [70]

$$\hat{H}_{so} = \frac{\alpha}{\hbar} \sum_i [\boldsymbol{\sigma}_i \times \mathbf{p}] \boldsymbol{\nu}, \quad (16)$$

where $\alpha = 0.6 \cdot 10^{-9} eVcm$ for holes with $m^* = 0.5m_0$ (m_0 is the free electron

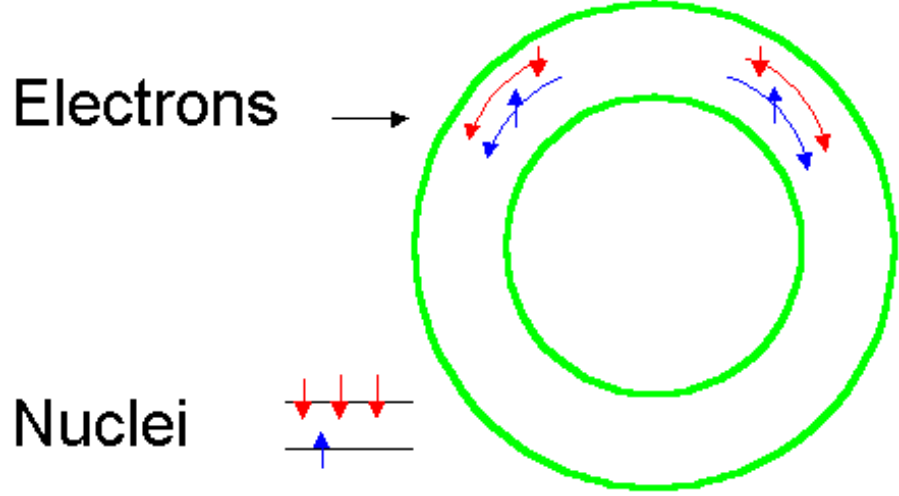


Figure 10: A mesoscopic ring with inhomogeneously polarised nuclear spins. The nonequilibrium nuclear spin population will result in Hyper - Aharonov - Bohm effect, as is described in the text. The left and right rotating electrons with both spin directions are shown.

mass), and $\alpha = 0.25 \cdot 10^{-9} eVcm$ for electrons, σ_i, \mathbf{p}_i are the charge carrier spin and momentum and ν is the normal to the surface. It can be rewritten in the form

$$\hat{H}_{so} = \mathbf{p} \mathbf{A}_{eff}, \quad (17)$$

where

$$A_{eff}^{GaAs} \simeq \frac{\alpha m^*}{\hbar} \langle \sigma \rangle, \quad (18)$$

$\langle \sigma \rangle$ stands for a nonequilibrium carriers spin population. Under the conditions of a topologically nontrivial orientation of \mathbf{A}_{eff}^{GaAs} the wave function of a charge carrier encircling the ring gains the phase shift similar to the one in an external magnetic field like in the ordinary ABE. This phase shift can be estimated as follows

$$2\pi\Theta = \frac{1}{\hbar} \oint A_{eff}^{GaAs} dl = \frac{m^*}{\hbar^2} \langle \sigma(t) \alpha \rangle \sim \frac{m^* \sigma(t) \alpha}{\hbar^2} L, \quad (19)$$

where L is the ring perimeter. To observe the oscillatory persistent current connected with the adiabatically slow time-dependent $\langle \sigma(t) \rangle$, L is supposed

to be less than the phase breaking length. Taking the realistic values for $L \approx 3\mu m$ and $\langle\sigma\rangle \approx 0.05 \div 0.1$ we estimate $2\pi\Theta \sim 5 \div 10$ which shows the experimental feasibility of this effect.

The standard definition of the spontaneous diamagnetic current is

$$j_{hfso} = -c \frac{\partial F}{\partial \phi} \Big|_{\phi_{ext}=0}, \quad (20)$$

where F is the electron free energy and ϕ_{ext} is the external (probe) magnetic flux. The oscillations of PC arise due to the exponential time dependence of the phase $\Theta_{eff}^0 \exp\{-t/T_1\}$ in Eq. (19), with the time constant T_1 . From the analogy with the standard ABE in a mesoscopic ring, we expect the following form for a persistent current in an one dimensional quantum ring at low enough ($T \ll \Delta$) temperature

$$j_{hfso} \sim \frac{ev_F}{L} \sin(2\pi\Theta_o e^{-\frac{t}{T_1}}). \quad (21)$$

Here Θ_o is the initial phase value. The marking difference between the periodical time dependence of standard Aharonov-Bohm oscillations, which are observed usually under the condition of linear time variation of the applied magnetic field and the hyperfine driven oscillations which die off due to the exponential time dependence of the nuclear polarization.

4.2 Nuclear spin polarization induced (NSPI) nano-structures

In [23] it was proposed to use the inhomogeneous hyperfine field to create so-called nuclear spin polarization induced (NSPI) magnetic structures, such as magnetic quantum dots, wires, rings, superlattices etc.

The spin splitting ($\mu_B B_{hf}$) due to a hyperfine magnetic field can be comparable to the Fermi energy of 2DEG, so that the electrons in the region, where nuclear spins are polarized, will occupy the energetically more favorable states with the spins opposite to \mathbf{B}_{hf} . The inhomogeneous nuclear polarization acts on the electrons as the effective confining potential $V_{conf} = -\mu_B B_{hf}$. This effective confining potential can be used to create different nanostructures with polarized electrons in them.

The method of local nuclear spin optical polarization allows to create different NSPI quantum structures (quantum dots, rings, wires, etc.) using the same sample and different illumination masks.

The time evolution of the hyperfine magnetic field in NSPI quantum structures, due to the nuclear spin relaxation and the nuclear spin diffusion, leads to variation of the number of transverse modes and corresponding

electron energies at a constant gate potential that can be directly measured by transport experiments.

The dependence of the conductance at 'zero' temperature on the number of transverse modes in the conductor is given by the Landauer formula $G = 2e^2MT/h$, where T is the average electron transmission probability, M is the number of the transverse modes and the factor 2 stands for the spin degeneracy. It is assumed that the transition probability T is independent of the energy in a small interval between the chemical potentials of the reservoirs. Usually, the number of the transverse modes, defined by the effective width of the conductor, is controlled by the gate voltage and the conductance is changed in discrete steps $2e^2/h$. It is underlined in [23] that due to the spin selective effective potential the height of the conductance steps is just e^2/h which is a half of the conductance quantum $G_0 = 2e^2/h$.

The experimental feasibility is based on the method of optical nuclear spin polarization [4, 43, 44, 84]. The resolution of the optical illumination of the sample can be high enough. Usual optic technique allows to create the light beams of the width of the order of the wave length ($\sim 500nm$), by using near fields optics the beam width can be sufficiently reduced ($\sim 100nm$). Hence a NSPI QW of the width of $1\mu m$ can be created by the modern experimental technique. In semiconductor heterostructures having supreme quality, the electron mean free path can be as large as $100\mu m$ and NSPI QW will operate in the quantum regime.

4.3 Hyper-anomalous Hall effect (HAHE)

In bulk semiconductors, the study of the conduction electron magnetotransport under the influence of the nonequilibrium nuclear spin polarization was started a long time ago by measurement of the magnetoresistance in bulk InSb [71]. Very recently a refined detailed study of the influence of a weak hyperfine field on the Hall effect in bulk InSb under a strong magnetic field (one occupied Landau level) was performed [56].

Apart of the usual Hall effect, caused by the drift of electrons in the crossed electric and magnetic fields, there exist also another contribution to the nondiagonal components of the conductivity tensor, the anomalous Hall effect [72, 73] (AHE), which is caused, in nonmagnetic semiconductors, by the spin-orbit interaction combined with the carrier magnetization.

It was proposed in [21] that in the strong nuclear spin polarization limit and at low magnetic field this effect may give a leading contribution to the anomalous Hall effect. Due to this interaction, electrons with their spin polarization parallel to the magnetization axis will be deflected at right

angles to the directions of the electric current and of the magnetization while electrons with antiparallel spin polarization will be deflected in the opposite direction. Thus, if the two spin populations are not equal there appears a net current in the transverse direction.

Until now, studies of the anomalous Hall effect have been limited to the case where the carrier magnetization is induced by the magnetic field. The magnetic field, however, produces a much larger normal Hall effect which makes experimental studies quite difficult.

Recording the Hall voltage in a sample with a dynamically polarized nuclear spins, during a time scale of the order of the nuclear spin relaxation time, which can be very long on the scale of typical electron equilibration processes, gives a direct measure of the spin dependent component of the Hall voltage. If the experiment is designed in such a way that the hyperfine field is much higher than the external magnetic field the hyperfine field driven anomalous Hall effect can be observed.

InSb is well suited for observation of the HAHE effect because of the strong spin-orbit coupling and the strong, due to the high atomic numbers of the atoms involved, hyperfine interaction ($\Delta = 0.9$ eV, $E_g = 0.235$ eV, $g^* = -51$, $m^* = 0.013m_0$). The maximal nuclear-spin polarization for all isotopes of InSb yields $B_{HF}^{\max} \approx 0.37$ T [21].

It is outlined in [21] that the HAHE gives the possibility to observe genuine spin dependent anomalous Hall effect which relies on the natural relaxation of nuclear spins and facilitates the experiment since the anomalous Hall voltage is not hidden by the much larger ordinary Hall effect.

4.4 Hyperfine residual resistance and the τ_ϕ problem

The possibility that the hyperfine interaction between the conduction electron spins and nuclear spins may result in hyperfine residual resistivity (HRR) in clean conductors at very low temperatures was studied theoretically by Dyugaev et al. [27]. Apart from the fundamental nature of this problem, the natural limitations on the mean free path are decisive in semiconductor based high speed electronic devices, like heterojunctions and quantum wells. The space periodicity of nuclei plays no specific role, as long as the nuclear spins are disordered and act as magnetic impurities [74] with the concentration $C_n \approx 1$. This scattering is not operative at extremely low temperatures, in the μK region when the nuclear spins are ferro- or antiferromagnetically ordered.

The HRR, arising due to the Fermi (contact) hyperfine interaction between the nuclear and the conduction electron spins can be written in atomic

units as:

$$V_{en} \approx Z\alpha^2 \frac{m_e}{M_n} Ry \quad (22)$$

Here $\hbar = m_e = e = 1$, m_e, M_n are the electron and the nucleon masses, respectively; $Ry = 27$ eV and $\alpha = 1/137$ is the fine structure constant.

In metals the effective electron-nuclear interaction constant is

$$g_n \equiv \frac{V_{ne}}{\epsilon_F} \approx 10^{-7} Z \frac{Ry}{\epsilon_F},$$

where the Fermi energy ϵ_F varies in a wide interval $0.01 \div 1$. g_n is 10^{-6} for *Li* and 10^{-3} for the rare earth metals.

The total residual resistivity is therefore a sum of the impurity $\rho_o(T \rightarrow 0) \sim C_o$ and the nuclear spin $\rho_n(T \rightarrow 0) \sim g_n^2$ contributions:

$$\rho_o^+(0^+) \approx \rho_{oo}(C_o + g_n^2). \quad (23)$$

Here $\rho_{oo} \approx 1$ in atomic units: $\rho_{oo} \approx 10^{-17}$ sec. The nuclear contribution to resistivity starts to be operative when the impurity concentration is $C_o \sim g_n^2$.

In the limit of an ideally pure ($C_o = 0$) metal the universal residual resistivity ρ_{URR} is, therefore $\rho_{URR} \geq \rho_{oo}g_n^2$ and the mean free path is limited by $10^{-8}/g_n^2$ cm. This yields 10^4 cm in *Li* and 10^{-2} cm for the rear earth metals. It is interesting to note that in materials with even-even nuclei (zero spin), like *Ca, Ni, Fe, Ce* and isotopically clean graphite *C*, where the electron-nuclear scattering is absent, the *HRR* would not be observed.

The temperature and magnetic field dependence of the *HRR* contains reach information on the hyperfine interaction between the conduction electrons and the nuclear spins. Usually, the temperature and the magnetic field dependence of residual resistivity due to nonmagnetic impurities is due mostly to the mesoscopic effects, and is vanishing in the limit $C_o \rightarrow 0$. In a magnetic field such that $\mu_e H \gg T$ the magnetic impurities freeze out and the Kondo effect is quenched. In order to freeze out the nuclear spins however one should apply much higher magnetic fields, $\mu_n H \gg T$. Therefore in the temperature interval $\mu_e H \gg T \gg \mu_n H$ the nuclear spin contribution may prevail even in metals with magnetic impurities.

In metals like *Li, Na, K, Rb, Cs, Au, Cu* the nuclear magnetic moments $I \neq 1/2$ and even without external magnetic field their $2I + 1$ degeneracy is lifted partially by the quadrupole effects (in the case of cubic crystal symmetry the quadrupole splitting of the nuclear levels may happen due to the defects [3, 75] and dislocations [76]).

While the normal metals have a quite similar electronic structure, the experimentally observed temperature dependence of the dephasing time τ_φ is quite different. This was shown in [77, 78], where the value of τ_φ was defined by the magnetoresistance measurements of long metallic wires *Cu*, *Au*, *Ag* in a wide temperature interval $10^{-2} < T < 10^\circ\text{K}$. In *Cu* and *Au* wires [77] τ_φ saturates at low temperatures which contradicts the standard theory [79]. Strangely enough the *Ag* wires do not show saturation [78] at the lowest temperatures, in accordance with [79].

It is conjectured in [22] that the influence of the quadrupole nuclear spin splitting on the phase coherence time τ_φ can be a clue to this puzzle. Indeed, the nuclear spins of both *Cu* and *Au* have a strong quadrupole moment ($s = 3/2$) and may act as inelastic two-level scatterers once their degeneracy is lifted by the static impurities [75], dislocations [76] and other imperfections. It is known that (see [80] and references therein) the nondegenerate two level scatterers may introduce inelastic phase breaking scattering of conduction electrons.

This may be not the case for *Ag* nuclei since their spin is $s = 1/2$. In this case the quadrupole splitting of nuclei spins by imperfections is negligible. In the absence of magnetic Zeeman splitting therefore the nuclear spins in *Ag* samples will act just as a set of elastic scatterers, and the temperature dependence of τ_φ should obey the standard theory [79].

4.5 Nuclear spins and superconducting order

The problem of coexistence of the superconducting and magnetic ordering, in spite of its long history [81], is still among the enigmas of modern condensed matter physics. Most of the theoretical and experimental efforts were devoted to studies of the coexistence of electron ferromagnetism and superconductivity. The possibility of a reduction of $H_c(T)$ by the nuclear ferromagnetism was outlined by Dyugaev et al. in [27] and theoretically studied in more details in [28, 30]. It was experimentally observed by several groups [58, 59].

It was recently conjectured [29] that the hyperfine part of the nuclear-spin-electron interaction may result in the appearance of a nonuniform superconducting order parameter, the so called Fulde-Ferrel-Larkin-Ovchinnikov state (FFLO) [82]. The FFLO state was thought originally to take place in superconductors with magnetically ordered magnetic impurities [82]. The main difficulty, however, in the observation of the FFLO caused by magnetic impurity ordering is in the simultaneous action of the "electromagnetic" and "exchange" parts of the magnetic impurities on the superconducting order.

In most of the known superconductors the "electromagnetic" part destroys the superconducting order before the "exchange" part modifies the BCS condensate to a nonuniform FFLO state.

The situation may change drastically in the case of nuclear spin ferromagnetic ordering. Indeed, the nuclear magnetic moment $\mu_n = \hbar e/M_i c$, is at least three orders of magnitude smaller than the electron Bohr magneton $\mu_e = \hbar e/m_o c$, so that the "electromagnetic" part of the nuclear spin fields is quite low, compared to that of the magnetic impurities. On the other hand the "exchange" part is strongly dependent on the nuclear charge Z .

5 Nuclear spins as qubits

5.1 Quantum Hall quantum computation

A growing number of models for electron and nuclear spin based memory cells [32] and quantum information processing (or Quantum Computing-QC) has been recently proposed [33]-[39]. We will concentrate in what follows on the models based on the manipulation of nuclear spins in bulk semiconductor [35], heterostructures [34, 37] and quantum wires and rings [39].

In [34] we have proposed a quantum computer realization based on hyperfine interactions between the conduction electrons and nuclear spins embedded in a two-dimensional electron system in the quantum-Hall effect. For modifications and improvements of this model see a recent review [38].

Maniv et al. [19] have suggested a following physical process of preparing a coherent state in QH ferromagnets. Let us assume that at time $t = -t_0 < 0$ the filling factor was tuned to a fixed value $\nu = \nu_0 \neq 1$ and then kept constant until $t = 0$. If $t_0 \gg T_2(\nu_0)$ then at $t = 0$ the nuclear spin system is in the ground state corresponding to the $2D$ electron system at $\nu = \nu_0$. Suppose that at time $t = 0$ the filling factor is quickly switched (i.e. on a time scale much shorter than $T_2(\nu_0)$) back to $\nu = 1$ so that the nuclear spin system is suddenly trapped in its instantaneous configuration corresponding to $\nu = \nu_0 \neq 1$. Thus the nuclear spins for a long time t ($\gg T_2(\nu_0)$) will find themselves almost frozen in the ground state corresponding to the 2DES at $\nu = \nu_0$, since $T_2(\nu = 1) \gg T_2(\nu_0)$.

The main idea behind this scenario stems from the experimental observation of a dramatic enhancement of the nuclear spin lattice relaxation rate $1/T_1$, and of a sharp decrease of the Knight shift in optically pumped NMR measurements, as the filling factor is shifted slightly away from $\nu = 1$. The prevailing interpretation of these closely related effects, associates them with the creation of skyrmions (or antiskyrmions) in the electron spin distribution

as the $2D$ electron system moves away from the quantum Hall ferromagnetic state at $\nu = 1$.

The off-diagonal long range magnetic order [67] existing in this state, corresponds to a complex order parameter which is coupled locally through the hyperfine interaction to the nuclear spins [19]. It is thus expected that the proposed manipulation of the nuclear spin system can be performed in a phase coherent fashion over a spatial region with size of the order of the skyrmion radius.

The latter effect is still sufficiently weak to enable the survival of a coherent state of a large number of qubits in the computer memory at $\nu = 1$. To find an upper bound for the size of such a memory let us consider N independent nuclear spins located at various sites \mathbf{r}_j in the quantum well. A number n , stored in the computer memory, corresponds to the direct product of N pure nuclear spin states $|n\rangle = |n_1\rangle \otimes |n_2\rangle \otimes \dots \otimes |n_N\rangle$, where $|n_j\rangle = \sum_{\sigma=\pm 1} \delta_{n_j,\sigma} |j, \sigma\rangle$, and $|j, \sigma\rangle$ is a nuclear state with spin projection σ located at the site \mathbf{r}_j . To be able to start a significant quantum computing process, however, a coherent superposition of such products, i.e. $|\psi\rangle = \sum_{n=1}^N \alpha_n |n\rangle$ (see e.g. [83]), should be prepared at time $t = 0$. This superposition may be represented more transparently for our purposes by the direct product of mixed spin up and spin down states, $|\psi(t=0)\rangle = \prod_{j=1}^N \otimes (\sum_{\sigma=\pm 1} \alpha_{j,\sigma} |j, \sigma\rangle)$, with the normalization $\sum_{\sigma} |\alpha_{j,\sigma}|^2 = 1$. The mixing is expected to take place most efficiently via the flip-flop processes with the electron spins during the manipulation period when $\nu \neq 1$.

Let us further assume that the initial state $\psi(0)$ is a completely coherent state, so that each of the numbers of length N has an equal probability. This is a typical state required in carrying out efficient quantum calculations [83]. In this state $|\alpha_{j,\sigma}|^2 = 1/2$ for any j and σ .

It is evident that due to the complete coherence of the initial state $\psi(0)$ the survival probability $P_\psi(t)$

$$P_\psi(t) = \left[\frac{1 + \text{Re } J(t)}{2} \right]^N \approx \exp \left\{ -\frac{1}{2} N [1 - \text{Re } J(t)] \right\} \approx e^{-\frac{1}{2} N \Gamma(t)}$$

depends only on the decoherence factor $J(t)$. The decay of $P_\psi(t)$ therefore follows $\exp[-\frac{1}{2} N \Gamma(t)]$, saturating at $\exp[-\frac{1}{2} N \eta \tilde{C}^2]$ for $t \gg \hbar/\varepsilon_{sp}$. Despite the much larger drop in the level of coherence, the time scale over which the coherence diminishes is the same as in the case of a single qubit.

5.2 Kane model

The most popular and intensively developed is the Kane model [35], where the qubits are the nuclear spin of the ^{31}P donor placed near to the surface of Si . The nuclear spins are manipulated by electric gates above the Si surface via the atomic electrons. The two-qubit interaction is provided by the overlapping outer electrons of two neighboring ^{31}P donors. The Kane model has stimulated detailed theoretical and experimental activity.

The strength of the hyperfine interaction is proportional to the probability density of the electron wavefunction at the nucleus. In semiconductors, the electron wavefunction extends over large distances through the crystal lattice. Two nuclear spins can consequently interact with the same electron, leading to electron-mediated or indirect nuclear spin coupling.

Voltages applied to metallic gates in a semiconductor device may be used to control the hyperfine interaction and, correspondingly, the electron-mediated interaction between the nuclear spins.

A quantum mechanical calculation proceeds by the precise control of three external parameters:

- i) gates above the donors control the strength of the hyperfine interactions and hence the resonance frequency of the nuclear spins beneath them;
- ii) gates between the donors turn on and off electron-mediated coupling between the nuclear spins;
- iii) a globally applied a.c. magnetic field B_{ac} flips nuclear spins at resonance.

Custom adjustment of the coupling of each spin to its neighbors and to B_{ac} enables different operations to be performed on each of the spins simultaneously. Finally, measurements are performed by transferring nuclear spin polarization to the electrons and determining the electron spin state by its effect on the orbital wavefunction of the electrons, which can be probed using capacitance measurements between adjacent gates.

An important requirement for a quantum computer is to isolate the qubits from any degrees of freedom that may lead to decoherence. If the qubits are spins on a donor in a semiconductor, nuclear spins in the host are a large reservoir with which the donor spins can interact. Consequently, the host should contain only nuclei with spin $I = 0$. This simple requirement unfortunately eliminates all *III-V* semiconductors as host candidates, because none of their constituent elements possesses stable $I=0$ isotopes. Group *IV* semiconductors are composed primarily of $I=0$ isotopes and can in principle be purified to contain only $I = 0$ isotopes.

The only $I = 1/2$ shallow (group *V*) donor in Si is ^{31}P . At sufficiently

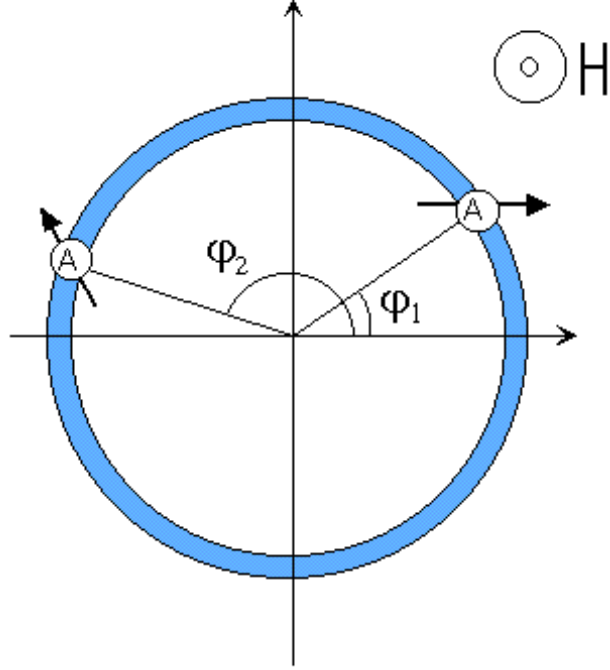


Figure 11: Two-spin-qubits in a nano-ring.

low ^{31}P concentrations at temperature $T = 1.5\text{ K}$, the electron spin relaxation time is thousands of seconds and the ^{31}P nuclear spin relaxation time exceeds 10 hours. It is likely that at millikelvin temperatures the phonon limited ^{31}P relaxation time is of the order of 10^{18} seconds [85], making this system ideal for quantum computation. Recent calculations of relaxation and decoherence times for this system are presented in [25].

5.3 Nuclear-spin qubits in nano-structures

In [39] a new system is proposed, which consists of nuclear spins (qubits) embedded into a mesoscopic ring, Fig. 11, or a finite length quantum wire, Fig. 12. The hyperfine interaction of the electrons in the system with nuclear spins leads to an effective indirect nuclear spin interaction

$$E = (I_{1,x}I_{2,x} + I_{1,y}I_{2,y}) A + I_{1,z}I_{2,z}B, \quad (24)$$

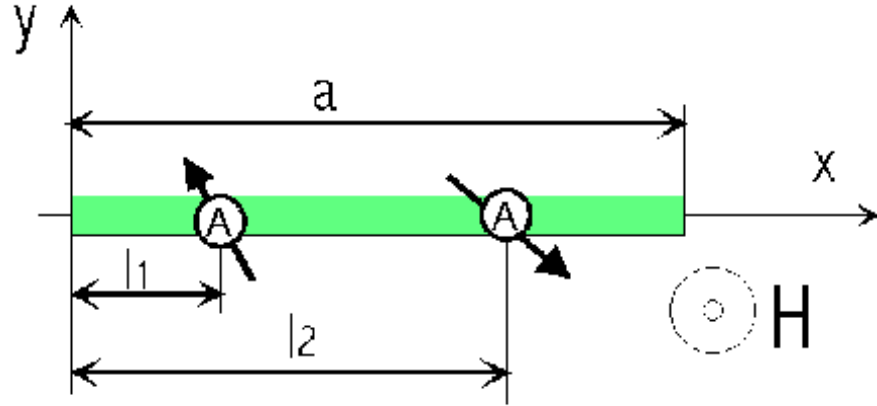


Figure 12: Two spin-qubits in a nano-wire.

where \vec{T}_i is magnetic moment of a nucleus, A and B are functions of the system parameters presented in Fig. 13. It is obtained in [39] that the effective nuclear spins interaction exhibits sharp maxima, Fig. 13, as function of the magnetic field and nuclear spin positions which opens the way to manipulate qubits with almost atomic precision. The selective nuclear spin interaction can be obtained by changing external parameters of the system, as is shown in Fig. 14.

In [39] the expression for indirect nuclear spin interaction between nuclear spins in a mesoscopic system, consisting of a finite length quantum wire or ring with nuclear spins placed in the zero nuclear spin matrix, is obtained. Interaction between any two qubits, which is necessary for two-qubit operations, is performed by the electrons in the wire. It is very sensitive to the system parameters: nuclear spin location, number of electrons, magnetic field and geometry of the system. Its dependence on the system parameters is completely different from indirect nuclear spin interaction in $2D$ and $3D$ metals: by varying the external parameters (magnetic field and number of electrons) one can control with almost atomic precision the nuclear spin interaction strength by creating maxima of the amplitude of electron wave function on some qubits and zero on the other. The connections to the measuring system, preparation of the initial state and performing of the one-qubit operations using the NMR could be similar to the existing experimental suggestions [36]. The decoherence time of the nuclear spins in mesoscopic systems is expected to be long enough to perform the quantum

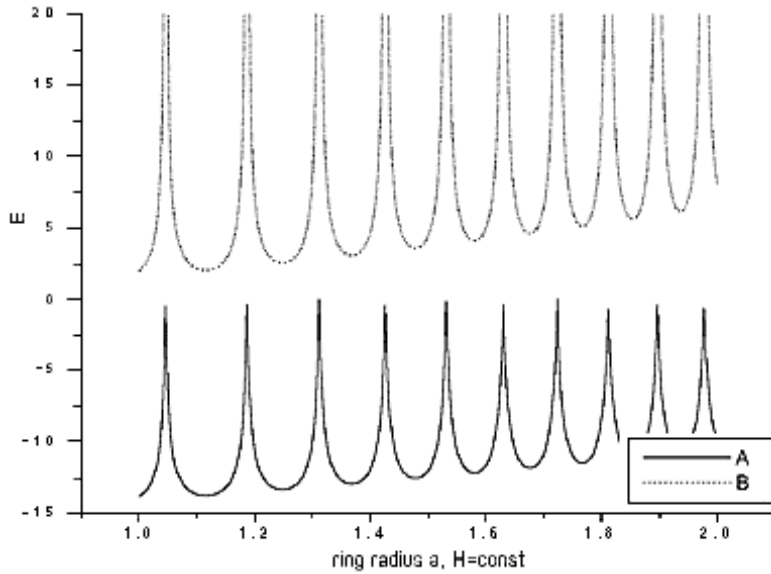


Figure 13: Interaction energy coefficients A and B, Eq. (24), as function of the magnetic flux through the ring.

computation, since the discrete electron spectrum in mesoscopic systems imposes restriction on the flip-flop processes and the nuclear spin relaxation time at low temperatures is expected to have an activation behavior.

6 Experiments

The measurement of the nuclear spin-relaxation in heterojunctions is a challenging experimental problem, since the direct detection of the NMR signals in solids requires usually $10^{17} - 10^{20}$ nuclei. The number of nuclear spins interacting with the two-dimensional electrons is however, much smaller: $10^{12} - 10^{15}$.

The first successful measurements of the magnetic field dependence of T_1^{-1} under QHE conditions were performed in a series of elegant experiments by the K. von Klitzing group, [40]. Combining electron-spin resonance (ESR) and resistivity measurement techniques they have observed the shifting of the ESR resonance frequency by the hyperfine field of nonequilibrium nuclear spin population, which is the well known Overhauser shift [3]. In this experiment the $2D$ electron Zeeman splitting is tuned to the pumping

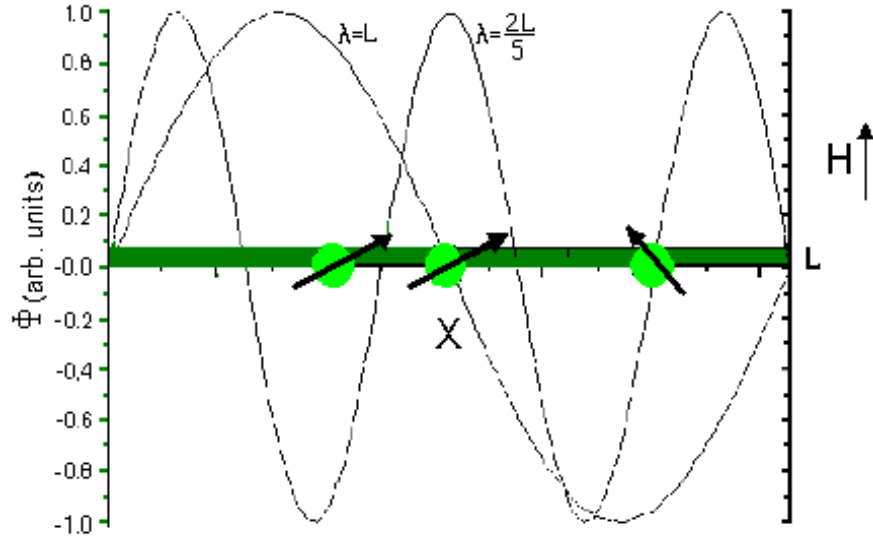


Figure 14: Indirect interaction via standing wave like conduction electron wave function between spin-qubits, embedded in a nano-wire of a finite length.

frequency. The angular momentum gained by a 2DEG electron, excited to the upper Zeeman branch, is then transferred to the nuclear spins, thus creating a nonequilibrium nuclear spin population.

These measurements show a close similarity between the magnetic field dependence of the nuclear spin-relaxation rate and the magnetoresistance in quantum Hall effect, as it was suggested theoretically in [8], thus demonstrating clearly the importance of the coupling of nuclear spins to the conduction electron spins in the nuclear relaxation processes in these systems.

Various experimental techniques were used since and in what follows we will describe shortly the main developments and achievements in experimental studies of the hyperfine coupling between the nuclear spins and the electrons in QHE, mesoscopic and superconducting systems.

Another way of measuring the nuclear spin relaxation and diffusion in a heterojunction under strong magnetic field by transport techniques (spin-diode) was demonstrated by Kane et al. [42]. They have reported measurements performed on “spin diodes“ : junctions between two coplanar 2DEG’s in which $\nu < 1$ on one side and $\nu > 1$ on the other. The Fermi level E_F crosses between spin levels at the junction. In such a device the 2DEG is highly

conducting except in a narrow region (with a width of the order of several hundred angstroms) where $\nu = 1$.

Wald et al. [54] presented the experimental evidence for the effects of nuclear spin diffusion and the electron-nuclear Zeeman interaction on interedge state scattering. Polarization of nuclear spins by dc current has proven to be a rich source for new, not yet understood completely, phenomena. This is the case, for example of anomalous spikes in resistivity around certain fractional filling factors, observed by Kronmuller et al. [55] and in resistively detected NMR in QHE regime reported by Desrat et al. [52]. Influence of nonequilibrium nuclear spin polarization on Hall conductivity and magnetoresistance was observed and studied in detail by Gauss et al. [56].

In 1994 Barrett et al. [43] observed, for the first time, a sharp NMR signal in multi-quantum wells, using the Lampel [84] technique of polarizing the nuclear spins by optical pumping of interband transitions with near-infrared laser light (*OPNMR*). Polarization of nuclei results in a significant enhancement of the NMR sensitivity, since the resonance in a two-level system results in equalizing their population. The difference in the population is, obviously, maximal when the spins are completely polarized.

Detailed studies of the Knight shift data suggested [43] that the usually accepted picture of electron spins, aligned parallel to the external field, should be modified to include the possibility of topologically nontrivial nuclear spin orientations, the Skyrmions. Optical polarization of nuclear spins was used also as a tool for reducing the Zeeman splitting of 2D electrons by Kukushkin et al. [47]. This resulted in a noticeable enhancement of the skyrmionic excitations. Similar results are reported by Vitkalov et al. [48].

Very recently, a modern ultra-sensitive NMR spin-echo technique was employed to study the physics of QHE [49] in GaAs/AlGaAs multi-quantum well heterostructures. The spin polarization of 2DES in the quantum limit was investigated and the experimental data support the noninteracting Composite Fermion model in the vicinity of the filling factor $\nu = 1/2$. Using the same technique the polarization of 2DES near $\nu = 2/3$ was investigated [50]. It was shown there that a quantum phase transition from a partially polarized to a fully polarized state can be driven by increasing the ratio between the Zeeman and Coulomb energies.

An amazing phenomenon, following from the hyperfine coupling between the electron and the nuclear spins, is the giant enhancement of the low temperature heat capacity of GaAs quantum wells near the filling factor $\nu = 1$, discovered in 1996 by Bayot et al. [45]. As other thermodynamic properties, it experiences quantum oscillations, following from the oscillatory density of states $D(E_F)$ at the Fermi level.

At about $T = 25mK$, and in clean samples, Bayot et al. [45] have observed anomalous deviations from the free electron model, in the specific heat value (up to four orders of magnitude) for the filling factor in the range $0.5 \leq \nu \leq 1.5$. Their explanation is that in this interval of parameters, the electron system couples strongly to nuclear spin system with a concentration of several orders of magnitude larger than the electron one.

This raises a question about the origin of the strong coupling between the electron and the nuclear spins in the interval $0.5 \leq \nu \leq 1.5$. The guess is the skyrmions, since they are predicted to appear just in the same interval of the filling factor. Additional support for this mechanism is in the results of [49], where the disappearance of the nuclear spin contribution to the heat capacity was reported, as the ratio between the Zeeman and Coulomb energies exceeds a certain critical value. The Zeeman splitting of electrons was modified in these experiments by tilting the magnetic field.

A new very promising technique for measuring spatially varying nuclear spin polarization within a GaAs sample is reported in [57]. In the force detected NMR (FDNMR) the sample is mounted on a microcantilever in an applied magnetic field. A nearby magnetic particle creates a gradient of magnetic field which exerts a force on the magnetized sample and triggers the cantilever oscillations. FDNMR is capable to perform the magnetic resonance imaging of the sample with a very high accuracy. This method can be useful in defining the spatial distribution of nuclear spin polarization in non homogeneous samples. This information may be crucial for understanding different peculiarities of data obtained by previously described methods.

A new world of the low-temperature physics of the hyperfine interactions in superconducting metals opens in the μK region, where the nuclear spins start ordering, thus reducing the critical magnetic field of superconductors as has been recently discovered by the Pobell group [58]. They have studied the magnetic critical field $H_c(T)$ of a metallic compound $AuIn_2$ where the superconductivity sets up at $T_{ce} = 0.207K$. They have observed, in $AuIn_2$, the nuclear spin ferromagnetic ordering at $T_{cn} = 35\mu K$. It was observed in these experiments that the magnetic critical field $H_{c0} = 14.5$ G is lowered by almost a factor of two at $T < T_{cn}$.

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