

CHAPTER 5. SPECIAL TOPICS IN POWER ELECTRONICS

Generator of current pulses having stable parameters, based on a nonlinear inductor obtained using switched linear capacitors and a gyrator

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Received 1 March 2005, accepted 6 June 2005

Abstract

A current-controllable nonlinear inductor with a simple-form but a strongly nonlinear (singular) flux-current characteristic, obtained by means of a switched capacitor unit and a gyrator, is proposed. The unit includes linear capacitors that are switched at some given voltage level(s). It is important for the point of nonlinearity that the moments of switchings are not given a priori, but are defined by the voltage on the switched unit, which is one of the state variables of the circuit. The use of linear elements, and the fact that the circuit does not include physical (magnetic) inductors, makes the circuit reliable and suitable for integrated implementation, which is relevant, in particular, for the realization of multi-element nonlinear circuits. An application of an oscillatory circuit, including such a nonlinear inductor, might also be the generation of current pulses having a stable amplitude. This might be important, e.g., for the operation of solid-state lasers. Numerical simulations of the inductor circuit are in agreement with theoretical propositions.

1 Introduction

In order to consider a singular-characteristic nonlinear inductor obtained by employing a capacitor unit, let us first refer to the analytical model in [1], where a smooth nonlinear inductance characteristic is created using a nonlinear *ferroelectric capacitor* [2,3] and a *gyrator* [1,4], as is schematically shown in Fig. 1.

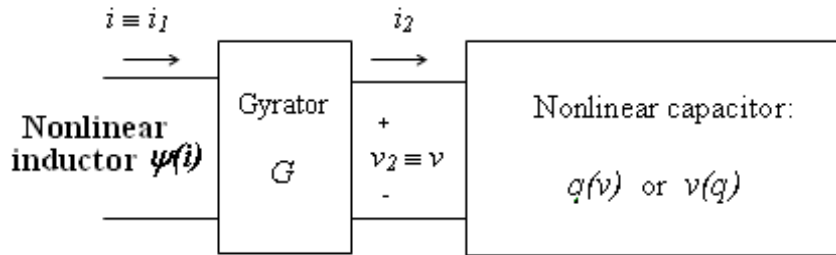


Figure 1: The schematic circuit for obtaining a nonlinear inductor. In [1] the ‘analytical’ nonlinear (in practice, ferroelectric) capacitor at the output of the gyrator is considered. Here we take a (nonlinear) *switched* capacitor circuit with a *piecewise-linear* characteristic $q(v)$ (or $v(q)$), of the type shown in Fig. 2. The characteristic $\psi(i)$ of the inductor, seen at the input of the gyrator, will also be found of such a form (Fig. 4).

It is shown in [1] that if the charge-voltage characteristic of the capacitor is $q = q(v)$, then the inductor’s equivalent ‘*flux*’-*current* characteristic realized by this circuit, is

$$\psi(i) = G^{-1}q(G^{-1}i). \quad (1)$$

In the linear case, when $q(v) = Cv$, (1) becomes

$$\psi(i) = G^{-2}Ci ,$$

and since in this case, $\psi(i) = Li$, we obtain the well-known linear relation between L and C , $L = G^{-2}C$, together with the known interpretation of the gyrator’s action as to turning a linear capacitor into a linear inductor (or conversely, an inductor into a capacitor, $C = G^2L$).

According to (1), gyrator turns the capacitor's characteristic $q(v)$, not necessarily linear, into the inductor's characteristic $\psi(i)$ keeping the form of the characteristic function. In terms of differential inductance $L(i)$ we obtain from (1)

$$L(i) = d\psi/di = G^{-1}dq(G^{-1}i)/di, \quad (2)$$

or, stressing the role of the differential capacitance in the modeling,

$$L(i) = G^{-2} \left. \frac{dq(v)}{dv} \right|_{v=G^{-1}i}. \quad (3)$$

The use of a ferroelectric capacitor possessing a smooth $q(v)$ is, however, *not practical* because of the possible breakdown of the capacitor at a voltage at which its nonlinearity is strongly exhibited, because of the analytical problematicity associated with the ferroelectric hysteresis [2], and because of the fact that ferroelectric-layer technology still does not fit the usual integrated circuit technology.

These problems may be avoided by replacing the ferroelectric capacitor in the circuit by a switched capacitor unit composed of *linear* capacitors of any suitable technological performance. Since the switching of the linear capacitors is done *at certain voltage levels*, we obtain a singular *nonlinear* $v - q$ characteristic of the capacitor (capacitor unit). Fig. 2 gives the most proper for our needs here example of the characteristic obtained by means of such a switched unit.

It is worth stressing that if the switching were to be done at prescribed moments, then the piecewise linear characteristic $q(v)$ would lead to a *linear* (time-dependent) circuit. This case would also be relevant to a generation of sharp spikes, but not to electronic modeling of singular nonlinear physical processes, which is another interesting application point. (See [7] for a relevant proposition.)

We shall use the simple continuous piecewise-linear $q(v)$ with one inflection point, using the symbols $C_{initial}$ and C_{final} for the initial and final differential capacitances. This will allow us to refer to the results of [3,5] associated with some specific stability of a voltage (below current) pulse. A circuit realizing $q(v)$ with more than one inflection point would require sequential (at the voltage scale) switching of more than two linear capacitors. For the pulse generator application, a characteristic with one inflection point is quite appropriate, and even can be optimal as regards obtaining maximal height of the pulse.

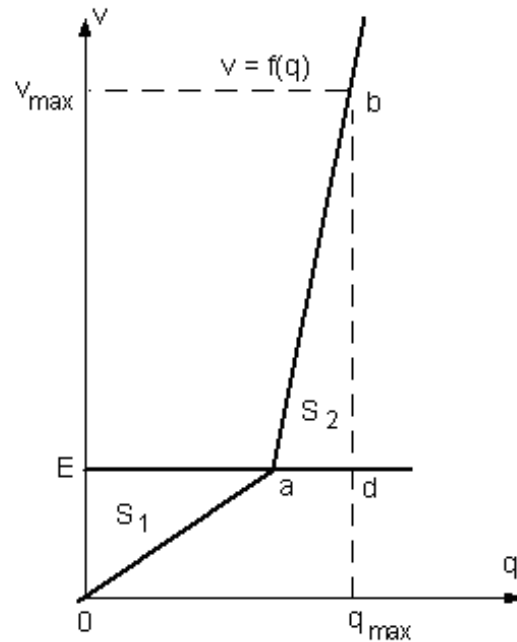


Figure 2: The singular characteristic of a nonlinear capacitive unit. Parameter E is taken from the circuit in Fig. 5. The switchings of the linear capacitor need not be done when the voltage on the unit equals E , but this is the optimal case [5], providing maximal amplitude (i.e. maximal value of v_{max}) of the transient voltage on the capacitor. The general case of switching is analyzed in [5].

2 The singular nonlinear inductor; the PSpice simulations

Since the mathematical relations [4] that define a gyrator are purely algebraic, the gyrator's action should not depend on whether the switching occurs at prescribed moments (when the whole system would be *linear* time-variant), or when the voltage on a capacitor reaches a prescribed level. Nevertheless, we would like to confirm experimentally that a gyrator maps $q(v) \rightarrow \psi(i)$ according to (1) for the *singular nonlinear* characteristics, stressing that the singular case is technologically the most promising, and the nonlinearity is important for some applications.

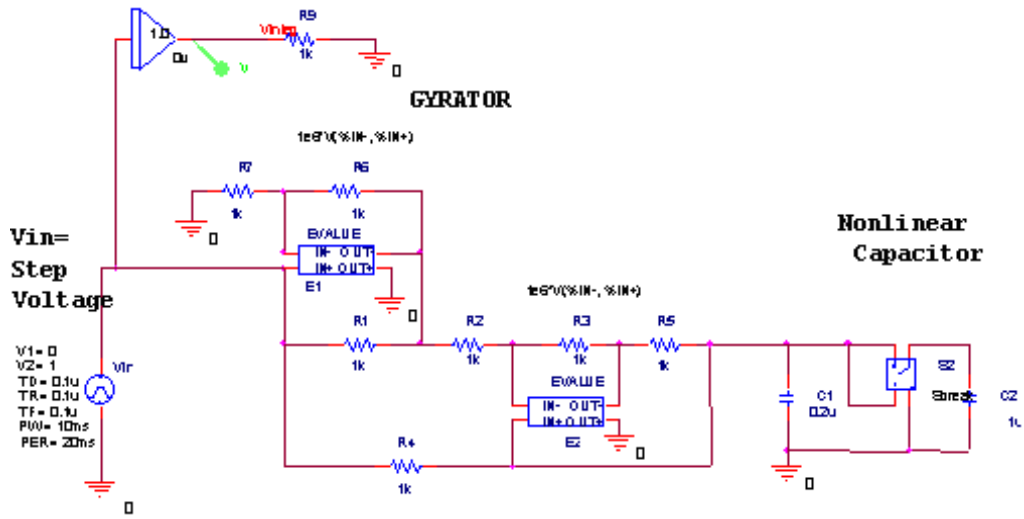


Figure 3: The PSpice realization of the gyrator loaded by the switched-capacitive circuit. The switch in the nonlinear capacitive unit conducts when the voltage v on C_1 is less than 0.8V, and is disconnected for higher voltages.

The PSpice circuit is shown in Fig. 3. The input step-voltage is of 1V height, and the gyrator has $G = 10^{-3}\Omega^{-1}$. The initial value, $C_{initial}$, of the capacitor unit is $C_1 + C_2 = 0.2\mu\text{F} + 1\mu\text{F} = 1.2\mu\text{F}$, and the final value, C_{final} , is $C_1 = 0.2\mu\text{F}$. The switching value of the capacitor's voltage v , when C_2 becomes disconnected (or connected again), is $v_s = 0.8\text{V}$, slightly smaller than $v_{in} = 1\text{V}$.

In the experiment, a constant input voltage leads to the input current increasing in time with a constant slope defined by the value of the capacitance at the output of the gyrator, which shows the inductive nature of the whole circuit; $v \sim di/dt$. This is simply explained by the gyrator action: the input voltage is transformed to the output current, $i_2 = Gv_1$, which continuously charges the capacitor, and the increasing capacitor's voltage $v \equiv v_2$ is transformed to the input current $i \equiv i_1 = Gv_2$.

In order to pass from $v_1 = d\psi/dt$ to the 'flux' ψ , we integrate (this is done in the same PSpice simulation) the input voltage over time, and thus finally obtain the characteristic $\psi(i)$ of the inductor.

This experimental characteristic $\psi(i)$ is shown in Fig. 4. We find that $L_{final}/L_{initial} = C_{final}/C_{initial} = 1/6$ with a good precision. We also have

$L_{initial} = G^{-2}C_{initial} = 1.2\text{H}$, and $L_{final} = G^{-2}C_{final} = 0.2\text{H}$.

The presence of an inflection point in $\psi(i)$ at $i = Gv_s$ agrees with the gyrator equation $i_1 = Gv_2$.

Thus the simulations confirm the assumption of the similarity of $\psi(\cdot)$ and $q(\cdot)$, predicted by (1), for the singular characteristic as well, and we have here a method for realizing nonlinear inductances by means of linear capacitive elements that are well implemented in integrated circuits.

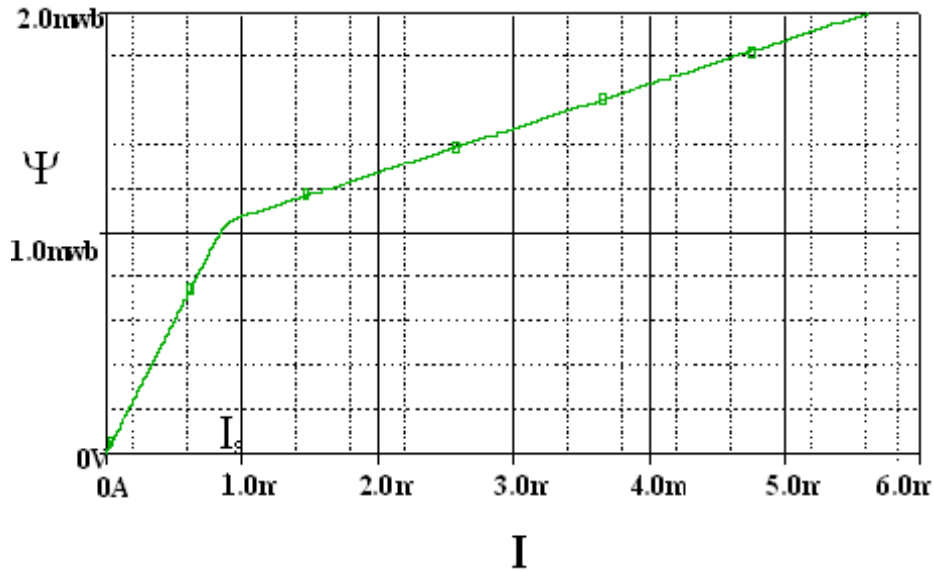


Figure 4: The piecewise-linear simulated $\psi(i)$ (i is in mA, ψ in mWb) of the nonlinear inductor, directly corresponding to $q(v)$ of the nonlinear capacitive unit at the output of the gyrator. $L_{initial} = 1\text{mWb}/0.83\text{mA} \approx 1.2\text{H}$ which equals to $G^{-2}C_{initial} = (10^6\Omega^{-2})(1.2\mu\text{F})$, and $L_{final}/L_{initial} = C_{final}/C_{initial}$, with a good precision.

3 On the possibility of creating a generator of current pulses of a stable amplitude

Turning to an oscillatory circuit, let us continue meanwhile without gyrator, i.e., with generation of *voltage* pulse. The dual circuit with the singular nonlinear inductor and *current* pulse is then immediate (Sect. 3.2).

Considering a process in an LC circuit, we use only the *first* oscillatory

spike appearing after the input stress. Though the oscillations decay as time passes, the influence of small losses on the height of the first single spike can be ignored. One can obtain such "first spikes" periodically by repeating the input stresses. Such periodically repeated transient oscillations are shown below in Fig. 7. In the context of suitability of the circuit for technological realization, it is very important that zero input current leads to a proper dependence of the spike's height on $C_{initial}$ and C_{final} .

3.1 Preliminary discussion of a voltage-spike generator

Employing the simple piecewise-linear $q(v)$ with one inflection point, we use, as before, the symbols $C_{initial}$ and C_{final} for the initial and final differential capacitances. The associated nonlinear switched capacitive unit with two linear capacitors had already found application [3,5] in an optimizing circuit for a voltage pulse generator. In the oscillatory circuit shown in Fig. 5, which includes the switched capacitive unit, the *maximal* value of the spike(s) amplitude, $(v(t))_{max}$, is *optimized by the choice of switching level of the nonlinear oscillations of the voltage on the capacitor unit*.

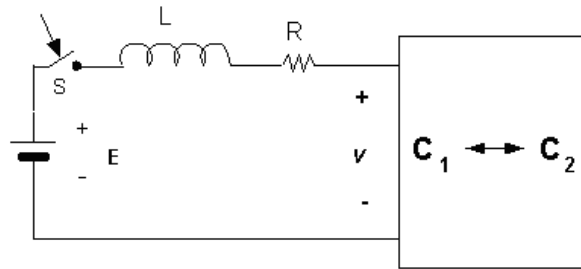


Figure 5: The oscillatory LC circuit with the switched capacitive unit having a characteristic as in Fig. 2, and step-type voltage input. The first sharp spike of the oscillatory $v(t)$ is as in Fig. 7 below. It is important that the process starts at zero current. This leads to the technologically desirable role of the ratio $C_{final}/C_{initial}$ in expression for the height of the spike given by (4).

The optimization of the transient amplitude v_{max} means obtaining its maximal value under conditions of zero initial input current and given $C_{initial}$ and C_{final} . Having the zero initial current may be a technical condition associated with starting the process by closing the series switch (see [3,5] for details).

The optimized value equals

$$[(v(t))_{max}]_{optimized} = [1 + (C_{initial}/C_{final})^{1/2}]v_o. \quad (4)$$

This maximal value for the amplitude is obtained by the switching $C_{initial} \rightarrow C_{final}$ precisely at the moment when the oscillating voltage on the capacitor unit reaches, while increasing, the level E (see Fig. 2).

Equation (4) follows from the equation of energy balance (consider Fig. 2 and see also [3,5]):

$$Eq_{max} = \int_0^{q_{max}} v(q) dq$$

which yields

$$\frac{C_{initial}E^2}{2} = \frac{C_{final}}{2}[v_{max} - E]^2. \quad (5)$$

For a linear circuit, setting in (4) $C_{initial} = C_{final}$, we obtain the well known result $v(t)_{max} = 2E$. In the strongly nonlinear case of $C_{initial} \gg C_{final}$, (4) gives much higher (stronger) pulses. The pulses of the strongly nonlinear oscillations of the $v(t)$ are sharp and well separated.

Fig. 6 shows a special circuit where a diode, or an SCR, helps one to realize/repeat such a transient process periodically, using the sinusoidal line voltage.

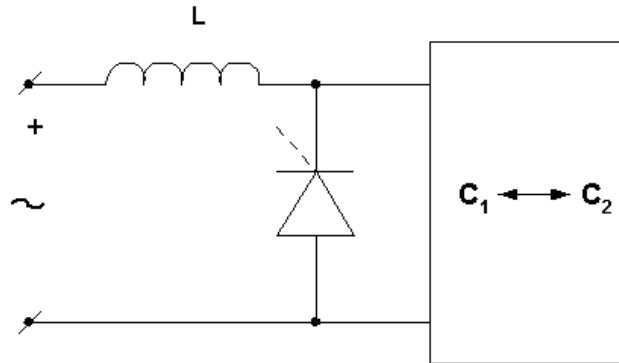


Figure 6: The circuit for periodic repetition (using the low-frequency line voltage) of the transient process in the oscillatory circuit of the type shown in Fig. 5 (see [5] for more details). Since inductor's voltage leads the current, when the diode stops conducting, the input voltage is maximal, and the oscillatory process in the L-C circuit starts, with the zero initial conditions, precisely as in Fig. 5.

Fig. 7 shows an associated oscilloscope picture with the real voltage oscillations.

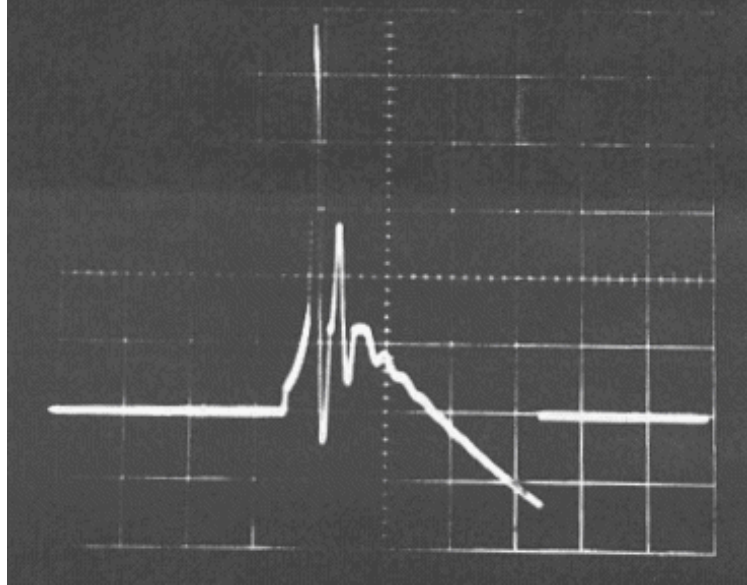


Figure 7: The sharp *voltage* pulses obtained in the circuit of the type shown in Fig. 6. We expect such *current* pulses to be obtained when transferring to a dual circuit which employs a gyrator that turns the switched capacitive circuit into a switched inductor one.

The fact of *nonlinearity-caused enhancing* of the spikes of the transient oscillatory process by the factor (term 2 relates to the case of a linear capacitor)

$$\frac{[1 + (C_{initial}/C_{final})^{1/2}] - 2}{2} = \frac{(C_{initial}/C_{final})^{1/2} - 1}{2},$$

is important for the application in [3], but it is even more important here that for similar technological performance of the capacitors, this factor depends on the capacitances *only via the ratio* $C_{final}/C_{initial}$ that by itself is weakly dependent on temperature and on parasitic capacitances necessarily present in the real circuit. Observe that this factor is obtained, in particular, due to using the transient process with initially zero current. The more usual interruption of the current in an inductor would lead to voltage spikes that are dependent on the value of the inductor and the capacitor's char-

acteristic, and though one could thus obtain a stronger voltage spike, the noted suitability of the technological performance would be lost.

All these features of the switched circuit remain in force and are important when we come to a dual circuit generating current pulses.

3.2 LC circuit including inductor with a singular nonlinear characteristic

Consider the oscillatory circuit in Fig. 8, which is *dual* to the circuit in Fig. 5. This circuit includes a regular capacitor, the nonlinear switched inductor unit, and a source of step current; $i_{input}(t) = i_o u(t)$, where $u(t)$ is the unit-step function, and i_o is the “level” of $i_{input}(t)$.

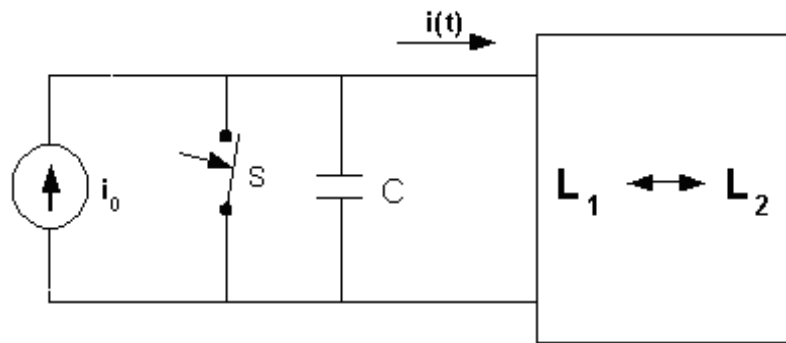


Figure 8: The circuit dual to the circuit in Fig. 5, implementing the switched subcircuit in Fig. 3. The spike is generated as follows. First, L is large, and most of the current enters the capacitor that accumulates significant energy. Then, L becomes small, and, in the transient oscillatory process, the capacitor’s energy is transferred to the small inductance. Since the magnetic energy $L_{final}i^2/2$ must thus reach a certain value, $i \sim (L_{final})^{-1/2}$. This leads to (6). Finally, the maximal height of the current spike is expressed (7) only via capacitor parameters, which is due to the modeling of the switched inductive unit by the switched capacitor unit, using gyrator.

Since the nonlinear inductor unit is realized by means of the gyrator and switched capacitive unit as in Figs. 1 and 3, there is, in fact, no inductors in this circuit, and the current spikes are obtained only by means of the charge storing elements.

The nonlinearity of the circuit is now associated with performing the switching of the “inductors” at certain value of the current $i(t)$ entering the

inductor unit.

We start with $L_{initial}$ (we assume that $L_{final} \ll L_{initial}$), and the optimal switching $L_{initial} \rightarrow L_{final}$ is done when $i(t) = i_o$. For such a switching,

$$[(i(t))_{max}]_{optimized} = [1 + (L_{initial}/L_{final})^{1/2}]i_o. \quad (6)$$

The derivation of (6) is similar to the derivation of (4), but in terms of the magnetic energies. However, in fact, this formula also follows from duality of the circuits in Figs. 5 and 8 (we ignore R , and other parameters are meant to be properly adjusted), i.e. from similarity of the associated dynamic circuit equations. That is, (6) directly follows from (4).

We can introduce in (6) the parameters $C_{initial}$ and C_{final} of the switched capacitive unit that imitates the switched inductive unit via gyrator. In view of the equality (Sect. 2) $L_{final}/L_{initial} = C_{final}/C_{initial}$, (6) simply becomes

$$[(i(t))_{max}]_{optimized} = [1 + (C_{initial}/C_{final})^{1/2}]i_o. \quad (7)$$

Thus, similarly to the voltage spikes shown in Fig. 7, we expect to obtain here high and sharp *current spikes* with the height of the first spike dependent on the ratio $C_{final}/C_{initial}$, according to (7).

Current pulses are generated, for instance, in the operation of solid-state lasers [6], and the stability of such pulses, following from the dependence of the pulse height on the ratio of the capacitors, will be important there. Perhaps, such an application can be relevant not only for the lumped version of the circuit, but also for some integrated implementations of the circuit.

4 Conclusions and final remarks

We have presented a gyrator-type realization of a strongly nonlinear inductor, *suitable for integrated circuit (or chip) implementation*. No physical inductor is involved.

According to the equality $i_1 = Gv_2$, a gyrator transforms a voltage function into a current function, here voltage spikes into current spikes. The moments of switching are defined by the development of the process in time, and are not given *a priori*. Thus, we have a nonlinear circuit.

Using the controllable sources included in gyrators, one reduces inductances in microelectronics to the much more technologically suitable capacitors, not only for linear but also for nonlinear circuits, and can create integrated circuits with many nonlinear elements, which may be of interest, in particular, for modeling some statistical physical processes, as is suggested in [7].

The main proposed application is the generation of a current pulse of stable amplitude for the operation of solid-state lasers.

Another interesting field for the application of such switched (either voltage or current, and with differing numbers of switching points per period) generators may be the generation of a pulse of a proper form at the input of a nonlinear “transmission line”, in order to initiate in the line a certain kind of solitary wave [8,9].

From the theoretical point of view, the nonlinearity of the type discussed is expressed, in a (dynamic) system description, by means of forms of the type $f(t - t_1)$, where $f(\cdot)$ may be found *a priori*, but t_1 is a zero-(level-) crossing of an initially unknown state variable (here, the capacitor’s voltage function $v(t)$). Using the terminology introduced in [7], such a term/form means a “zero-crossing nonlinearity”. Such nonlinearity is typical for some very basic physical processes, which should be considered for electronic modeling of these processes. We thus see the present discussion in the generalizing contexts of both [1] and [7].

I am grateful to Mr. Adir Zvulon for the PSpice simulations.

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