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Analysis of Energy Factor and mathematical modeling for power DC-DC converters

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Abstract

Mathematical modelling of power DC/DC converters is a historical problem accompanying development of the DC/DC conversion technology since 1940's. The traditional mathematical modelling is successful to describe fundamental converters but not available for complex structure converters due to a dramatic increase of the order of corresponding differential equations. We have to search an other way to establish mathematical modelling for power DC/DC converters.

Energy storage in power DC/DC converters has been paid attention to since long time ago. Unfortunately, there is no clear concept to describe the phenomena and reveal the relationship between the stored energy and the characteristics of power DC/DC converters. In this paper, we have theoretically defined a new concept - *Energy Factor* (EF) and investigated the relations between EF and the mathematical modelling of power DC/DC converters. EF is a new concept in power electronics and conversion technology, which thoroughly differs from the traditional concepts such as power factor (PF), power transfer efficiency (η), total harmonic distortion (THD), and ripple factor (RF). EF and the subsequential other parameters can illustrate the system stability, reference response, and interference recovery. This investigation is very helpful for system design, and DC/DC converters characteristics.

Keywords: mathematical modeling, energy factor (EF), power factor (PF), power transfer efficiency (η), total harmonic distortion (THD), ripple factor (RF), power DC/DC converters, system stability, step response and impulse response.

1 Introduction

Mathematical modelling of power DC/DC converters is a historical problem accompanying development of the DC/DC conversion technology since 1940's. Many experts such as Sira-Ramirez, Czarkowski, Ilic, Lee, Cuk and Middlebrook devoted in this area [1-8]. The traditional mathematical modelling is successful in describing fundamental converters, but not available for complex structure converters due to a dramatic increase of the order of corresponding differential equations. Fundamental DC/DC converters have been derived from choppers. The preliminary work on the mathematical modelling of power DC/DC converters followed the traditional calculation manner using impedance analysis to write a transfer function in the s-domain (Laplace transform). We have to search other ways to establish mathematical modelling of power DC/DC converters.

Energy storage in power DC/DC converters has been paid attention to since long time ago. Unfortunately, there is no clear concept to describe the phenomena and reveal the relationship between the stored energy and the characteristics of power DC/DC converters. In this paper, we have theoretically defined a new concept - *Energy Factor* (EF) and researched the relations between EF and the mathematical modelling for power DC/DC converters. EF is a new concept in power electronics and conversion technology, which thoroughly differs from the traditional concepts such as power factor (PF), power transfer efficiency (η), total harmonic distortion (THD), and ripple factor (RF). EF and the subsequential EF_V (and EF_{VD}) can illustrate the system stability, reference response, and interference recovery. This investigation is very helpful for system design and prediction of DC/DC converters characteristics. Two DC/DC converters: Buck converter and Super-Lift Luo-Converter are analysed as examples to demonstrate the applications of EF, EF_V, PE, SE, VE, time constant τ and damping time constant τ_d.

2 Second-order transfer function

A typical second-order transfer function in the s-domain is shown below:

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{M}{1 + s\tau + \xi s^2\tau^2} \tag{1}$$

where M is the voltage transfer gain, τ the time constant, τ_d the damping time constant ($\tau_d = \xi\tau$).

2.1 Very small damping time constant

If the damping time constant is very small (i.e. $\tau_d << \tau$, $\xi << 1$) and can be ignored, the value of the damping time constant τ_d is omitted (i.e. $\tau_d = 0$, $\xi = 0$). The transfer function (1) is downgraded from the second-order to the first order as

$$G(s) = \frac{M}{1 + s\tau}. \tag{2}$$

This is the transfer function of the first-order inertia element. This expression describes the characteristics of the DC/DC converter as a first-order inertia element. The step function response in the time-domain is

$$g(t) = M(1 - e^{-\frac{t}{\tau}}). \tag{3}$$

The transient process (settling time) is nearly 3 times of the time constant, 3τ, to produce $g(t) = g(3\tau) = 0.95M$. The response waveform in the time-domain is shown in Fig. 1 with $\tau_d = 0$.

The impulse interference response in the time-domain is

$$\Delta g(t) = U \cdot e^{-\frac{t}{\tau}} \tag{4}$$

where U is the interference signal. The interference recovering progress is nearly 3 times of the time constant, 3τ. The response waveform in the time-domain is shown in Fig. 2 with $\tau_d = 0$.

2.2 Small damping time constant

If the damping time constant is small (i.e. $\tau_d < \tau/4$, $\xi < 0.25$) but cannot be ignored, the value of the damping time constant τ_d is not omitted. The transfer function (1) restores its second-order character with two real poles σ_1 and σ_2 as

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{M/\tau\tau_d}{(s + \sigma_1)(s + \sigma_2)} \tag{5}$$

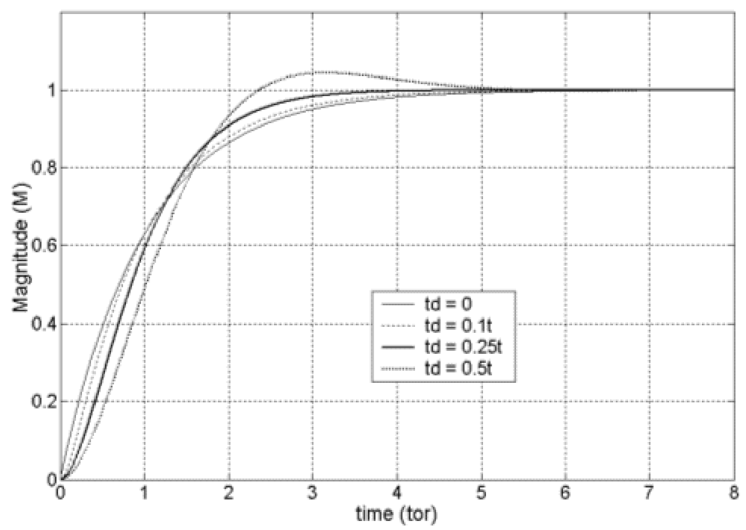


Figure 1: Step function responses ($\tau_d = 0$, 0.1τ, 0.25τ and 0.5τ).

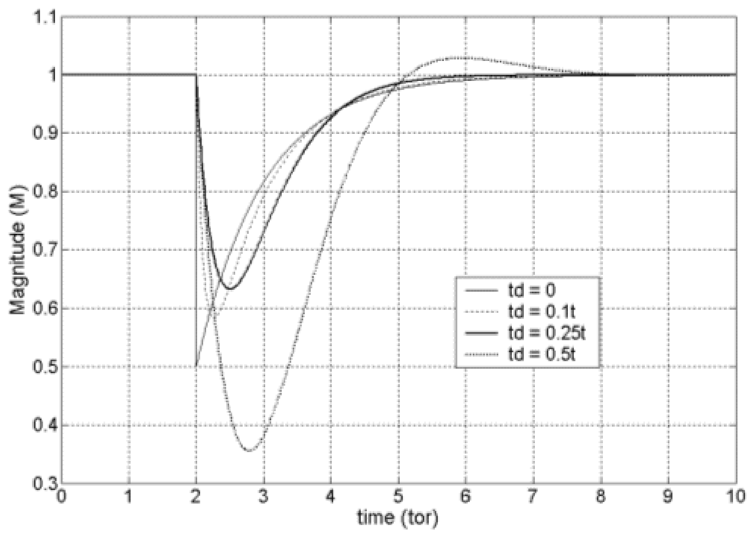


Figure 2: Impulse responses ($\tau_d = 0$, 0.1τ, 0.25τ and 0.5τ).

where

$$\sigma_1 = \frac{\tau + \sqrt{\tau^2 - 4\tau\tau_d}}{2\tau\tau_d} \text{ and } \sigma_2 = \frac{\tau - \sqrt{\tau^2 - 4\tau\tau_d}}{2\tau\tau_d}.$$

There are two real poles in the transfer function, and $\sigma_1 > \sigma_2$. This expression describes the characteristics of the DC/DC converter. The step function response in the time-domain is

$$g(t) = M\left(1 + K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}\right) \tag{6}$$

where

$$K_1 = -\frac{1}{2} + \frac{\tau}{2\sqrt{\tau^2 - 4\tau\tau_d}} \text{ and } K_2 = -\frac{1}{2} - \frac{\tau}{2\sqrt{\tau^2 - 4\tau\tau_d}}.$$

The transient process is nearly 3 times of the time value $1/\sigma_1$, $3/\sigma_1 < 3\tau$. The response process is quick without oscillation. The corresponding waveform in the time-domain is shown in Fig. 1 with $\tau_d = 0.1\tau$.

The impulse interference response in the time-domain is

$$\Delta g(t) = \frac{U}{\sqrt{1 - 4\tau_d/\tau}}\left(e^{-\sigma_2 t} - e^{-\sigma_1 t}\right) \tag{7}$$

where U is the interference signal. The transient process is nearly 3 times of the time value $1/\sigma_1$, $3/\sigma_1 < 3\tau$. The response waveform in time-domain is shown in Fig. 2 with $\tau_d = 0.1\tau$.

2.3 Critical damping time constant

If the damping time constant is equal to the critical value (i.e. $\tau_d = \tau/4$), the transfer function (1) is still of the second order with two equal real poles $\sigma_1 = \sigma_2 = \sigma$ as

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{M/\tau\tau_d}{(s+\sigma)^2} \tag{8}$$

where

$$\sigma = \frac{1}{2\tau_d} = \frac{2}{\tau}.$$

There are two folded real poles in the transfer function. This expression describes the characteristics of the DC/DC converter. The step function response in the time-domain is

$$g(t) = M\left[1 - \left(1 + \frac{2t}{\tau}\right)e^{-\frac{2t}{\tau}}\right]. \tag{9}$$

The transient process is nearly 2.4 times of the time constant τ, 2.4τ. The response process is quick without oscillation. The response waveform in the time-domain is shown in Fig. 1 with $\tau_d = 0.25\tau$.

The impulse interference response in the time-domain is

$$\Delta g(t) = \frac{4U}{\tau} t e^{-\frac{2t}{\tau}} \tag{10}$$

where U is the interference signal. The transient process is still nearly 2.4 times of the time constant, 2.4τ. The response waveform in the time-domain is shown in Fig. 2 with $\tau_d = 0.25\tau$.

2.4 Large damping time constant

If the damping time constant is large (i.e. $\tau_d > \tau/4$, $\xi > 0.25$), the transfer function (1) is a second-order function with a couple of conjugated complex poles s_1 and s_2 in the left-hand half plane in the s-domain as

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{M/\tau\tau_d}{(s+s_1)(s+s_2)} \tag{11}$$

where

$$s_1 = \sigma + j\omega \text{ and } s_2 = \sigma - j\omega,$$
$$\sigma = \frac{1}{2\tau_d} \text{ and } \omega = \frac{\sqrt{4\tau\tau_d - \tau^2}}{2\tau\tau_d}.$$

There is a couple of conjugated complex poles s_1 and s_2 in the transfer function. This expression describes the characteristics of the DC/DC converter. The step function response in the time-domain is

$$g(t) = M[1 - e^{-\frac{t}{2\tau_d}}(\cos\omega t - \frac{1}{\sqrt{4\tau_d/\tau - 1}} \sin\omega t)] \tag{12}$$

The transient response has an oscillating character with the damping factor σ and frequency ω. The corresponding waveforms in the time-domain are shown in Fig. 1 with $\tau_d = 0.5\tau$ and in Fig. 3 with τ, 2τ, 5τ and 10τ.

The impulse interference response in the time-domain is

$$\Delta g(t) = \frac{U}{\sqrt{\frac{\tau_d}{\tau} - \frac{1}{4}}} e^{-\frac{t}{2\tau_d}} \sin(\omega t) \tag{13}$$

where U is the interference signal. The recovery process is a curve with damping factor σ and frequency ω. The response waveforms in the time-domain are shown in Fig. 2 with $\tau_d = 0.5\tau$ and in Fig. 4 with τ, 2τ, 5τ, and 10τ.

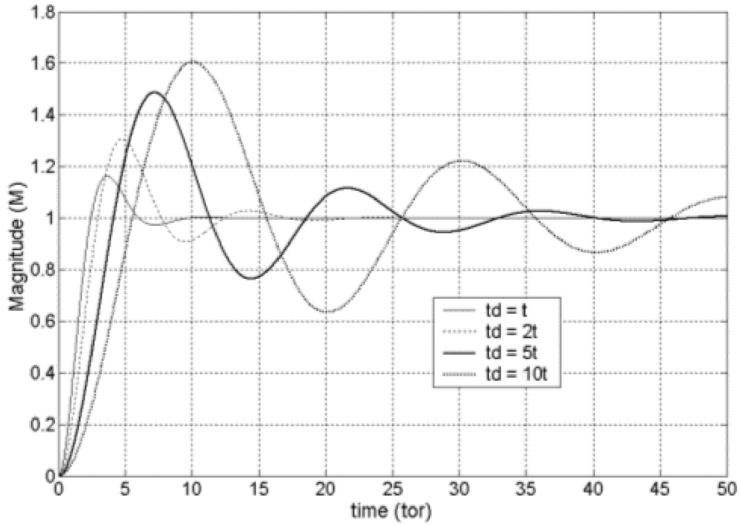


Figure 3: Step function responses ($\tau_d = \tau$, 2τ, 5τ, and 10τ).

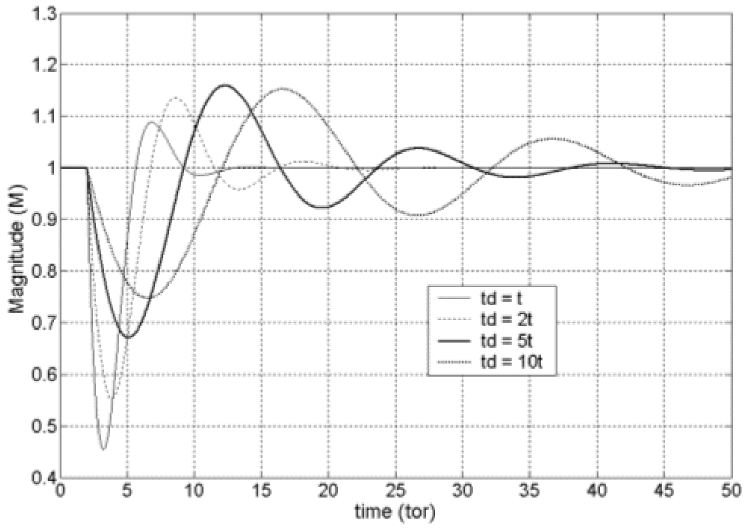


Figure 4: Impulse responses ($\tau_d = \tau$, 2τ, 5τ, and 10τ).

3 Traditional modeling for fundamental converters

Fundamental converters such as Buck converter in Fig. 5 (a), Boost converter in Fig. 5 (b), and Buck-Boost converter in Fig. 5 (c), consist of one inductor L and one capacitor C with the load R, and have the transfer function given in [9, 10]. For convenience, the input voltage and current are defined V_1 and I_1, and the output voltage and current are defined V_2 and I_2. The switching frequency is f, and the period $T = 1/f$. The conduction duty cycle is k.

$$G(s) = \frac{M}{1 + s\frac{L}{R} + s^2 LC} = \frac{M}{1 + s\tau + s^2 \tau\tau_d} \tag{14}$$

where M is the voltage transfer gain $M = V_2/V_1 = k$, τ the time constant $\tau = L/R$, τ_d the damping time constant $\tau_d = RC = \xi\tau$, s the Laplace operator in the s-domain.

It is a second-order transfer function in the s-domain. The corresponding dynamic equation is a second-order differential equation. This mathematical model is available for the case with **no** power losses during the conversion process. It was successfully used to describe the characteristics of a Buck converter: stability, transient process, step response (settling time), and impulse response (interference recovering time).

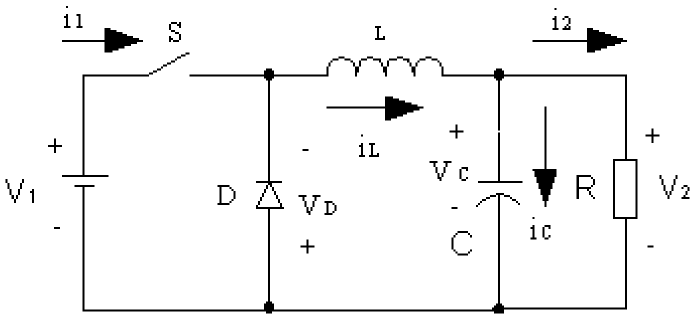
3.1 Mathematical modeling of a buck converter without power losses

A Buck converter shown in Fig. 5 (a) has the following values of the components: $V_1 = 40$ V, $L = 250$ μH, $C = 60$ μF, $R = 10$ Ω, the switching frequency $f = 20$ kHz ($T = 1/f = 50$ μs) and conduction duty cycle $k = 0.4$. Therefore, we have got the voltage transfer gain $M = k = 0.4$, i.e. $V_2 = kV_1 = 16$ V, the time constant $\tau = L/R = 25$ μs and the damping time constant $\tau_d = RC = 600$ μs. From cybernetic theory, this Buck converter is stable since the two poles (-s_1 and -s_2) are located in the left-hand half plane (LHHP):

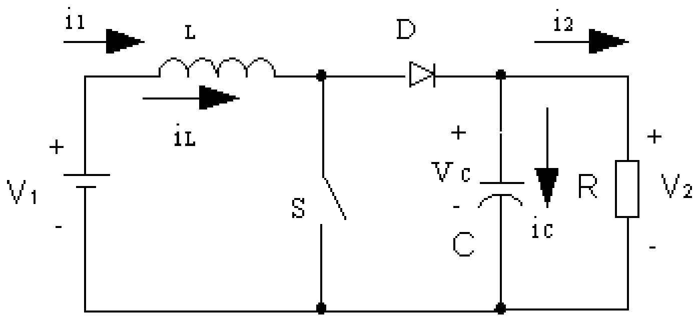
$$G(s) = \frac{M}{1 + s\tau + s^2 \tau\tau_d} = \frac{M/\tau\tau_d}{(s + s_1)(s + s_2)} \tag{15}$$

where

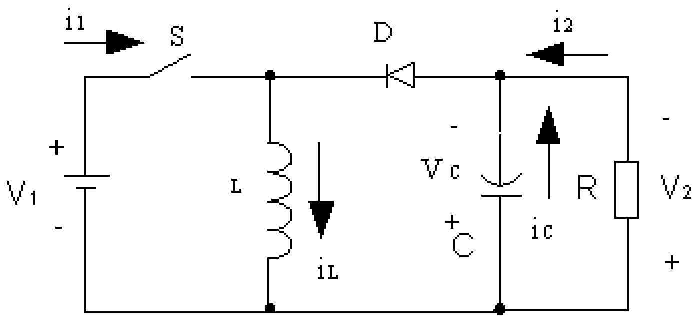
$$s_1 = \sigma + j\omega \text{ and } s_2 = \sigma - j\omega$$



(a) Buck converter



(b) Boost converter



(c) Buck-Boost converter

Figure 5: Fundamental converters.

with

$$\sigma = \frac{1}{2\tau_d} = \frac{1}{1200\mu s} = 833.33 \text{ Hz} \tag{16}$$

and

$$\omega = \frac{\sqrt{4\tau\tau_d - \tau^2}}{2\tau\tau_d} = \frac{\sqrt{60000 - 625}}{30000\mu} = \frac{243.67}{30000\mu} = 8122 \text{ rad/s.} \tag{17}$$

There is a couple of conjugated complex poles s_1 and s_2 in the transfer function. This expression describes the characteristics of the DC/DC converter. The step function response in the time-domain is

$$V_2(t) = 16[1 - e^{-\frac{t}{0.0012}}(\cos 8122t - 0.1026 \sin 8122t)] \text{ V.} \tag{18}$$

The step function response (transient process) has an oscillation character with the damping factor σ and frequency ω, and is shown in Fig. 6.

The impulse interference response in the time-domain is

$$\Delta V_2(t) = 0.205Ue^{-\frac{t}{0.0012}} \sin 8122t \tag{19}$$

where U is the interference signal. The impulse response (interference recovery process) has an oscillation character with the damping factor σ and frequency ω, and is shown in Fig. 7.

3.2 Mathematical modeling of a buck converter with small power losses

If there are some power losses in the form of a resistance r_L in the inductor L, we have got the following transfer function

$$G(s) = \frac{\frac{R}{R+r_L}k}{1 + s\frac{L+RCr_L}{R+r_L} + s^2LC\frac{R}{R+r_L}} = \frac{pk}{1 + s\tau + s^2\tau\tau_d} \tag{20}$$

where M is the voltage transfer gain

$$M = V_2/V_1 = pk,$$

τ the time constant

$$\tau = \frac{L + RCr_L}{R + r_L},$$

τ_d the damping time constant

$$\tau_d = \frac{LRC}{L + RCr_L},$$

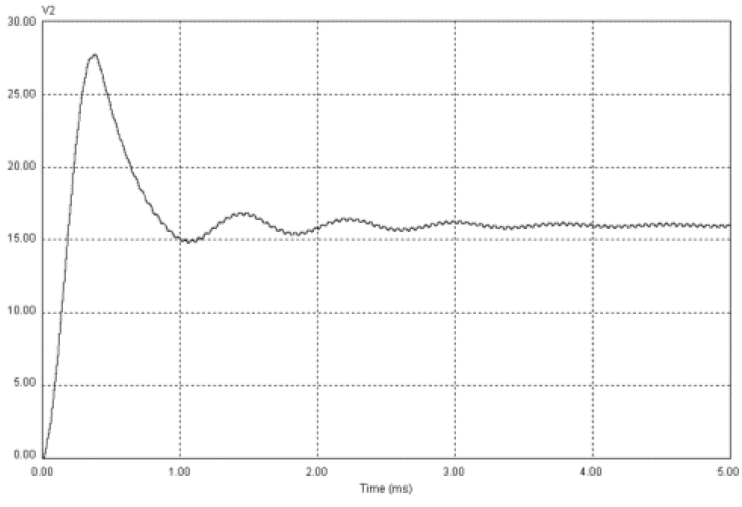


Figure 6: Step function response of a Buck converter without power losses.

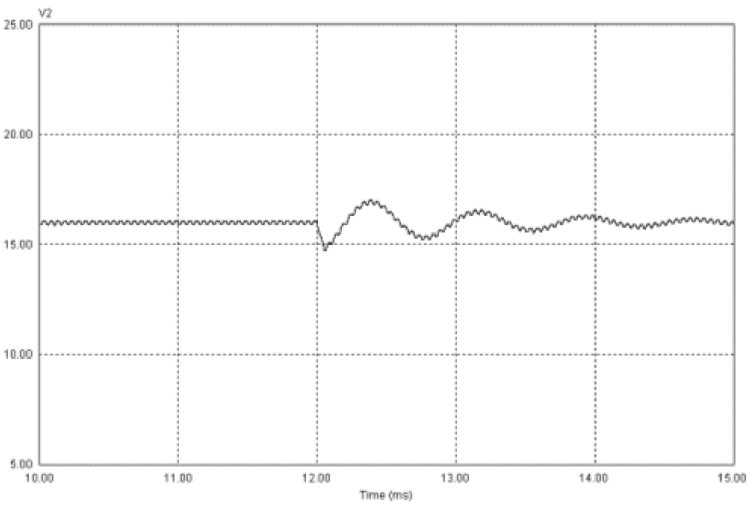


Figure 7: Impulse response of a Buck converter without power losses.

p the proportionality constant

$$p = \frac{R}{R + r_L}.$$

If the resistance r_L is equal to zero meaning no power losses, $p = 1$. To verify the correction of this mathematical model, we take the value of the resistance $r_L = 1.5\ \Omega$ with other parameters unchanged, which gives:

$$\tau = \frac{L + RCr_L}{R + r_L} = \frac{250\mu + 15 * 60\mu}{11.5} = 100\mu\text{s},$$

$$\tau_d = \frac{LRC}{L + RCr_L} = \frac{250\mu * 10 * 60\mu}{250\mu + 15 * 60\mu} = \frac{150000\mu}{1150} = 130.4\mu\text{s},$$

$$p = \frac{R}{R + r_L} = 0.87, \quad M = pk = 0.348.$$

Therefore,

$$\tau_d = 130.4\mu = 1.304\tau, \quad \xi = \frac{\tau_d}{\tau} = 1.304 > 0.25,$$

$$V_2 = pkV_1 = 0.87 * 0.4 * 40 = 13.9 \text{ V}.$$

This transfer function in the s-domain is still a second-order function. Since $\tau_d = 1.304\tau > 0.25\tau$, this Buck converter is stable and the two poles (-s_1 and -s_2) are located in the left-hand half plane (LHHP):

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{pk/\tau\tau_d}{(s + s_1)(s + s_2)} \tag{21}$$

where

$$s_1 = \sigma + j\omega \text{ and } s_2 = \sigma - j\omega$$

with

$$\sigma = \frac{1}{2\tau_d} = \frac{1}{260\mu s} = 3846 \text{ Hz} \tag{22}$$

and

$$\omega = \frac{\sqrt{4\tau\tau_d - \tau^2}}{2\tau\tau_d} = \frac{\sqrt{52000 - 10000}}{26000} = \frac{204.94}{26000} = 7882 \text{ rad/s}. \tag{23}$$

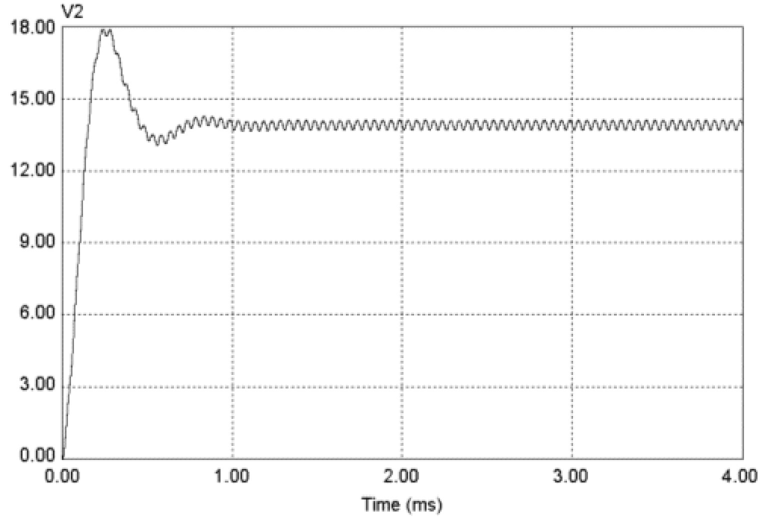


Figure 8: Step function response of a Buck converter with $r_L = 1.5\ \Omega$.

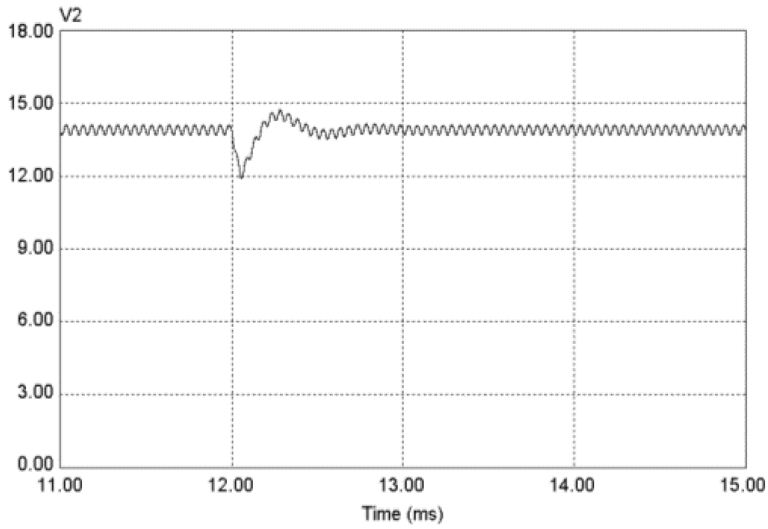


Figure 9: Impulse response of a Buck converter with $r_L = 1.5\ \Omega$.

The step function response in the time-domain is

$$V_2(t) = 13.9[1 - e^{-\frac{t}{0.00026}}(\cos 7882t - 0.487 \sin 7882t)] \text{ V}. \tag{24}$$

The step function response (transient process) has an oscillation character with the damping factor σ and frequency ω and is shown in Fig. 8.

The impulse interference response in the time-domain is

$$\Delta V_2(t) = 0.974Ue^{-\frac{t}{0.00026}} \sin 7882t \tag{25}$$

where U is the interference signal. The impulse response (interference recovery process) has an oscillation character with the damping factor σ and frequency ω, and is shown in Fig. 9.

3.3 Mathematical modeling of a buck converter with large power losses

If the value of the resistance $r_L = 4.5\ \Omega$ with other parameters unchanged, we have:

$$\tau = \frac{L + RCr_L}{R + r_L} = \frac{250\mu + 45 * 60\mu}{14.5} = 203.45\ \mu\text{s},$$

$$\tau_d = \frac{LRC}{L + RCr_L} = \frac{250\mu * 10 * 60\mu}{250\mu + 45 * 60\mu} = \frac{150000\mu}{2770} = 50.85\ \mu\text{s},$$

$$p = \frac{R}{R + r_L} = 0.69.$$

Therefore,

$$\tau_d = 203.45\mu = 0.24994\tau, \quad \xi = \frac{\tau_d}{\tau} = 0.24994 < 0.25,$$

$$V_2 = pkV_1 = 0.69 * 0.4 * 40 = 11.04 \text{ V}.$$

This transfer function in s-domain is still a second-order function. Since $\tau_d < 0.25\tau$, this Buck converter is stable and the two poles (-σ_1 and -σ_2) are real numbers located in the left-hand half plane (LHHP):

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{pk/\tau\tau_d}{(s + \sigma_1)(s + \sigma_2)} \tag{26}$$

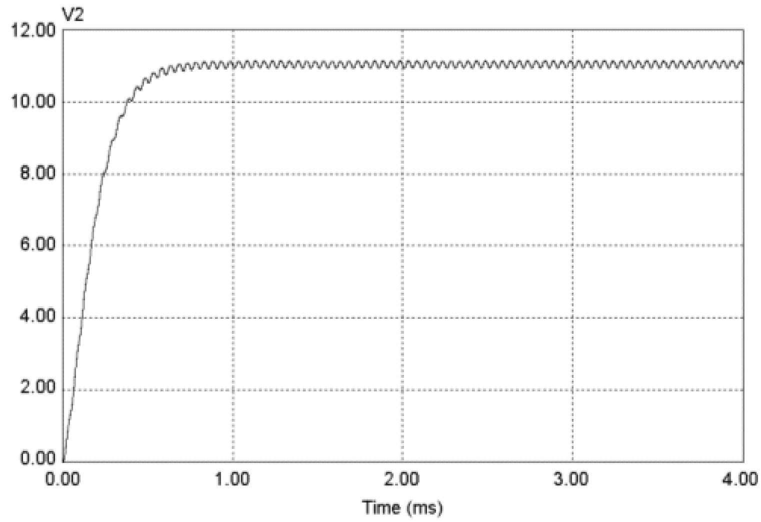


Figure 10: Step function response of a Buck converter with $r_L = 4.5\ \Omega$.

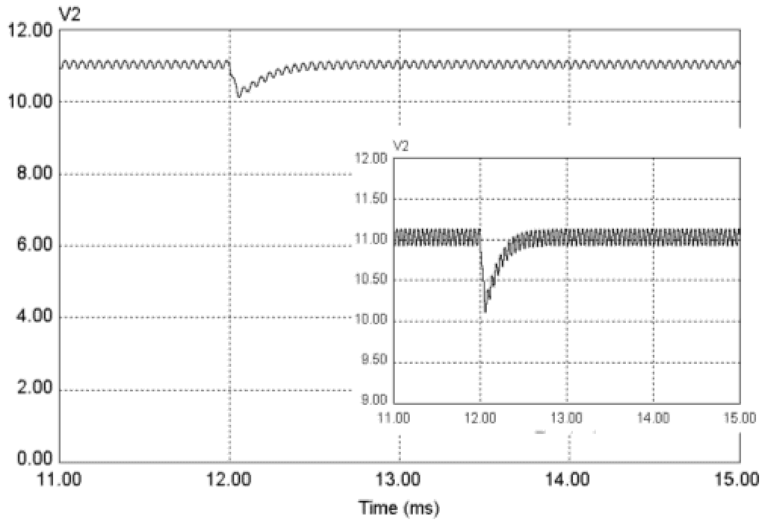


Figure 11: Impulse response of a Buck converter with $r_L = 4.5\ \Omega$.

where

$$\sigma_1 = \frac{\tau + \sqrt{\tau^2 - 4\tau\tau_d}}{2\tau\tau_d} = \frac{203.45 + \sqrt{41392 - 41382}}{20691\mu} = \frac{203.45 + 3.16}{20691\mu} = 9986 \tag{27}$$

and

$$\sigma_2 = \frac{\tau - \sqrt{\tau^2 - 4\tau\tau_d}}{2\tau\tau_d} = \frac{203.45 - \sqrt{41392 - 41382}}{20691\mu} = \frac{203.45 - 3.16}{20691\mu} = 9680. \tag{28}$$

The step function response in the time-domain is

$$K_1 = -\frac{1}{2} + \frac{\tau}{2\sqrt{\tau^2 - 4\tau\tau_d}} = -0.5 + \frac{203.45}{2\sqrt{41392 - 41382}} = -0.5 + 32.1 = 31.6,$$

$$K_2 = -\frac{1}{2} - \frac{\tau}{2\sqrt{\tau^2 - 4\tau\tau_d}} = -0.5 - \frac{203.45}{2\sqrt{41392 - 41382}} = -0.5 - 32.1 = -32.6,$$

$$V_2(t) = MV_1(1 + K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}) = 11\left(1 + 31.6e^{-9986t} - 32.6e^{-9680t}\right) \text{ V}. \tag{29}$$

The step function response (transient process) has no oscillation character and is shown in Fig. 10.

The impulse interference response in the time-domain is

$$\Delta V_2(t) = \frac{U}{\sqrt{1 - 4\tau_d/\tau}}\left(e^{-\sigma_2 t} - e^{-\sigma_1 t}\right) = 63.8U\left(e^{-9680t} - e^{-9986t}\right) \text{ V} \tag{30}$$

where U is the interference signal. The impulse response (interference recovery process) has no oscillation character, and is shown in Fig. 11.

3.4 Remarks

This mathematical model (17) is available for Boost converter and Buck-Boost converter as well. It is very easy to perform the operations and calculations in Sect. 3.1 – 3.3 for Boost converter and Buck-Boost converter. However, it is difficult to use this method to complex structure converters such as Luo-Converters, Cuk converter and SEPIC since they contain more inductors and capacitors in those converters so that their transfer functions are of a forth or higher order.

From application practice, most experts guessed that a second-order transfer function is good enough to describe the characteristics of power DC/DC converters. A popular way is the order-reducing method. If some inductors or/and capacitors have very large values, their current or/and voltage variations are small and can be ignored. For example, the Super-Lift Luo-Converter shown in Fig. 12 has the following transfer function (without power losses):

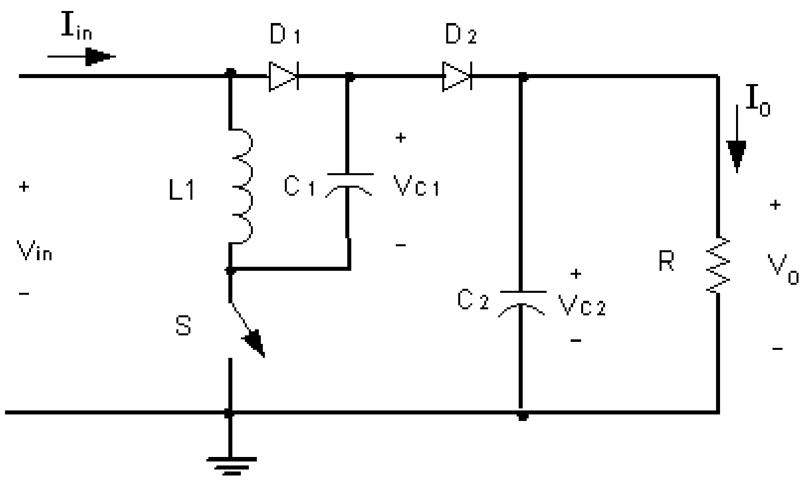


Figure 12: Super-Lift Luo-Converter.

$$G(s) = \frac{M\frac{R}{1+sC_2R}}{sL + \frac{1}{sC_1} + \frac{R}{1+sC_2R}} = \frac{MsC_1R}{1 + s(C_1 + C_2)R + s^2LC_1 + s^3LC_1C_2R} \tag{31}$$

where $M = \frac{2-k}{1-k}$ is the voltage transfer gain with k, the conduction duty cycle. It is a third-order transfer function. If we choose C_1 much larger than C_2, i.e. $C_1 \gg C_2$ or $C_2/C_1 = 0$, it is downgraded to a second-order transfer function:

$$G(s) = \frac{M\frac{R}{1+sC_2R}}{sL + \frac{1}{sC_1} + \frac{R}{1+sC_2R}} = \frac{M}{1 + s\frac{L}{R} + s^2LC_2}. \tag{32}$$

Similarly, if some power losses, such as inductor's resistance r_L not equal to zero, we obtain the equation similar to (20) in Sect. 3.2:

$$G(s) = \frac{M}{1 + s\frac{L+RC_2 r_L}{R+r_L} + s^2 LC_2 \frac{R}{R+r_L}} = \frac{M}{1 + s\tau + s^2 \tau\tau_d} \tag{33}$$

where $M = p\frac{2-k}{1-k}$, and p is a proportionality constant.

This method can be sufficiently accurate for circuit analysis. Unfortunately, some industrial applications cannot satisfy the condition $C_1 \gg C_2$. It is more difficult to apply this method to some complex structure converters. For example, a positive-output Luo-converter has two inductors and two capacitors [9]. The conditions: $L_1 \gg L_2$ and $C_1 \gg C_2$ have to be selected for applying this order-reducing method. We have to find an other way to establish mathematical modeling of power DC/DC converters.

4 Energy factor and new mathematical modeling

Many traditional parameters such as power factor (PF), power transfer efficiency (η), total harmonic distortion (THD), and ripple factor (RF), have been successfully applied in power electronics and conversion technology [9–13]. Using these parameters, one can successfully describe the system characteristics. Power DC/DC converters usually possess DC input and DC output. Consequently, some parameters such as PF and THD are not available to describe the characteristics of power DC/DC converters.

Energy storage in power DC/DC converters has been paid attention to since long time ago [11]. Unfortunately, there is no clear concept how to describe the phenomena and reveal the relationship between the stored energy and the characteristics of power DC/DC converters [12]. In this paper, we have theoretically defined a new concept - *Energy Factor* (EF), and investigated the relations between EF and the mathematical modeling for power DC/DC converters. EF is a new parameter in power electronics and DC/DC conversion technology, which noticeably differs from the traditional concepts such as PF, efficiency η, THD, and RF. *Energy Factor* and the subsequential parameters: Energy Factor for variation of stored energy (EF_V and EF_{VD}), pumping energy (PE), stored energy (SE), variation of stored energy (VE) in continuous conduction mode (CCM) and variation of stored energy (VE_D) in discontinuous conduction mode (DCM), can illustrate the system stability, reference response, and interference recovery. This investigation is very helpful for system design and prediction of DC/DC converters characteristics.

4.1 Continuous conduction mode (CCM)

All power DC/DC converters have pumping circuit to transfer the energy from the source to some energy-storage passive elements, e.g., inductors and capacitors. The pumping energy (PE) is used to count the input energy during the switching period T. Its calculation formula is:

$$PE = \int_0^T V_1 i_1(t)dt = V_1 I_1 T. \tag{34}$$

The stored energy in an inductor is $W_L = \frac{1}{2}LI_L^2$, the stored energy across a capacitor is $W_C = \frac{1}{2}CV_C^2$.

Therefore, if there are n_L inductors and n_C capacitors, the total stored energy in a DC/DC converter is

$$SE = \sum_{j=1}^{n_L} W_{Lj} + \sum_{j=1}^{n_C} W_{Cj}. \tag{35}$$

The most powerful DC/DC converters consist of inductors and capacitors. Therefore, we define the capacitor-inductor stored energy ratio (CIR) [12]:

$$CIR = \frac{\sum_{j=1}^{n_C} W_{Cj}}{\sum_{j=1}^{n_L} W_{Lj}}. \tag{36}$$

Another factor is the *energy losses* in a period T, $EL = P_{loss} \times T$. We can define the efficiency η to be

$$\eta = \frac{PE - EL}{PE}. \tag{37}$$

The current flowing through an inductor has variations (ripple) Δi_L causing variations of stored energy in an inductor

$$\Delta W_L = \frac{1}{2}L(I_{L-\max}^2 - I_{L-\min}^2) \tag{38}$$

$$= L\frac{I_{L-\max} + I_{L-\min}}{2}(I_{L-\max} - I_{L-\min}) = LI_L\Delta i_L \tag{39}$$

where $I_{L-\max} = I_L + \Delta i_L/2$ and $I_{L-\min} = I_L - \Delta i_L/2$.

The voltage across a capacitor has variations (ripple) $\Delta \mathrm{v}_C$, variations of stored energy across a capacitor

$$\Delta W_C = \frac{1}{2}C(V_{C-\max}^2 - V_{C-\min}^2) \tag{40}$$

$$= C\frac{V_{C-\max} + V_{C-\min}}{2}(V_{C-\max} - V_{C-\min}) = CV_C\Delta v_C \tag{41}$$

where $V_{C-\max} = V_C + \Delta v_C/2$ and $V_{C-\min} = V_C - \Delta v_C/2$.

In the steady state of CCM, the total variation of the stored energy is

$$VE = \sum_{j=1}^{n_L} \Delta W_{Lj} + \sum_{j=1}^{n_C} \Delta W_{Cj}. \tag{42}$$

4.2 Discontinuous conduction mode (DCM)

In the steady state of DCM, some of the minimum current and/or voltage values become zero. We define the filling coefficients m_L and m_C to describe the discontinuous situation. Usually, if the switching frequency f is high enough, the inductor's current has a triangular waveform. It increases and reaches I_{max} during the switching-on period kT, and decreases during the switching-off period $(1-k)T$. It becomes zero at $t = t_1$ before next switching-on in DCM. The waveform is shown in Fig. 13 (a). The time t_1 should be $kT < t_1 < T$, and the filling coefficient m_L is

$$m_L = \frac{t_1 - kT}{(1-k)T} \tag{43}$$

where $0 < m_L < 1$. It means the inductor's current can only fill the time period $m_L(1-k)T$ during the switch-off. In this case, I_{min} is equal to zero and the average current I_L

$$I_L = I_{\max}[m_L + (1 - m_L)k] \tag{44}$$

and

$$\Delta i_L = I_{\max}. \tag{45}$$

Therefore,

$$\Delta W_L = LI_L\Delta i_L = LI_{\max}^2[m_L + (1 - m_L)k]. \tag{46}$$

We define, analogously, the filling coefficient m_C to describe the capacitor voltage discontinuity. The waveform is shown in Fig. 13 (b). Time t_2 should be $kT < t_2 < T$, and the filling coefficient m_C is

$$m_C = \frac{t_2 - kT}{(1-k)T} \tag{47}$$

where $0 < m_C < 1$. It means that the capacitor's voltage can only fill the time period $m_C(1-k)T$ during the switch-off. In this case, V_{min} is equal to zero and the average voltage V_C is

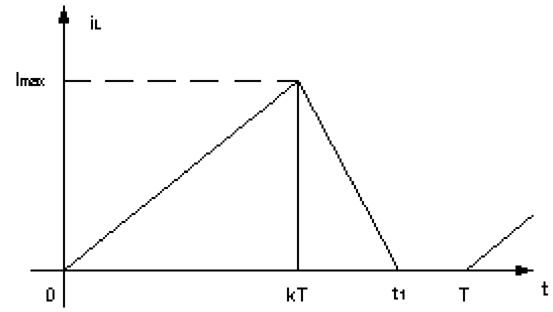
$$V_C = V_{\max}[m_C + (1 - m_C)k] \tag{48}$$

and

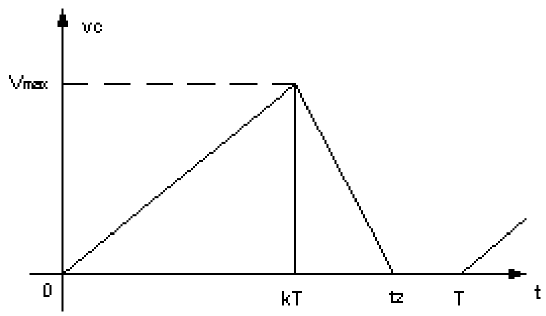
$$\Delta v_C = V_{\max}. \tag{49}$$

Therefore,

$$\Delta W_C = CV_C \Delta v_C = CV_{\max}^2[m_C + (1 - m_C)k]. \tag{50}$$



(a) Discontinuous inductor current



(b) Discontinuous capacitor voltage

Figure 13: Discontinuous inductor current and capacitor voltage.

We consider a converter working in DCM, which usually means that only one or two, but not all storage elements have a voltage/current discontinuity. We use the parameter VE_D to present the total variation of the stored energy:

$$VE_D = \sum_{j=1}^{n_{L-d}} \Delta W_{Lj} + \sum_{j=n_{L-d}+1}^{n_L} \Delta W_{Lj} + \sum_{j=1}^{n_{C-d}} \Delta W_{Cj} + \sum_{j=n_{C-d}+1}^{n_C} \Delta W_{Cj} \quad (51)$$

where n_{L-d} is the number of discontinuous inductor current, and n_{C-d} is the number of discontinuous capacitor voltages. We have other papers to discuss these cases. This formula form is same as equation (42). For convenience, we use equation (42) to cover both CCM and CDM except for some cases of a special necessity.

4.3 Energy Factor

The input energy in a period T is $PE = P_{in} \times T = V_1 I_1 \times T$. We now define the *Energy Factor* (EF) as the ratio of stored and pumping energy:

$$EF = \frac{SE}{PE} = \frac{SE}{V_1 I_1 T} = \frac{\sum_{j=1}^{m} W_{Lj} + \sum_{j=1}^{n} W_{Cj}}{V_1 I_1 T}. \quad (52)$$

We also define the *Energy Factor for the variation of stored energy* (EF_V) as the ratio of the variation of stored energy and pumping energy:

$$EF_V = \frac{VE}{PE} = \frac{VE}{V_1 I_1 T} = \frac{\sum_{j=1}^{m} \Delta W_{Lj} + \sum_{j=1}^{n} \Delta W_{Cj}}{V_1 I_1 T} \quad (53)$$

Energy Factor EF and variation Energy Factor EF_V can be used to describe the characteristics of power DC/DC converters. The applications are listed in the next sections.

5 Applications of the parameters

5.1 Power efficiency η

We can use these parameters to describe the characteristics of DC/DC converters. Usually most analysis applied in DC/DC converters assume the input power to be equal to the output power, $P_{in} = P_o$ or $V_1 I_1 = V_2 I_2$, so that pumping energy is equal to output energy in a period $PE = V_1I_1T = V_2I_2T$. It corresponds to the efficiency $\eta = V_2I_2T/PE = 100\%$. If the load is a pure resistive one, R, $V_2 = I_2R$, the voltage transfer gain of a DC/DC converter is

$$M = \frac{V_2}{V_1} = \frac{I_2R}{V_1}. \tag{54}$$

Particularly, power losses always exist during the conversion process. They are caused by the resistance of the connection cables, resistance of the inductor and capacitor wire, and power losses across the semiconductor devices (diode, IGBT, MOSFET and so on). We can divide them into the resistance power losses P_r, passive element power losses P_e and device power losses P_d. The total power losses

$$P_{loss} = P_r + P_e + P_d. \tag{55}$$

Therefore,

$$P_{in} = P_O + P_{loss} = P_O + P_e + P_e + P_d = V_2I_2 + P_e + P_e + P_d. \tag{56}$$

So that $P_{in} > P_o$ and the efficiency $\eta = V_2I_2T/PE < 100\%$. If the load is a pure resistive one, R, $V_2 = \sqrt{P_OR} = \sqrt{\eta P_{in}R}$, the voltage transfer gain of a DC/DC converter is

$$M = \frac{V_2}{V_1} = \frac{\sqrt{\eta P_OR}}{V_1} \tag{57}$$

5.2 System stability

After investigation we have found that all existing power DC/DC converters are stable, and have the condition $EF > EF_V$. If $EF \leq EF_V$, it means that variation is reaching 100% or higher, and the converter intends to be unstable.

5.3 Time constant τ of a power DC/DC converter

The *time constant* τ of a DC/DC converter is a new concept to describe the transient process of a DC/DC converter. In the presence of power losses it is defined as:

$$\tau = \frac{2T \times EF}{1 + CIR}(1 + CIR\frac{1-\eta}{\eta}) = \frac{2}{1 + CIR}\frac{SE}{V_1I_1}(1 + CIR\frac{1-\eta}{\eta}). \tag{58}$$

This time constant is independent of the switching frequency f (or period $T = 1/f$). It can be used to estimate the converter responses for a step function and impulse interference.

5.4 Damping time constant τ_d of a power DC/DC converter

The *damping time constant* τ_d of a DC/DC converter is a new concept to describe the transient process of a DC/DC converter. In the presence of power losses it is defined as:

$$\tau_d = \frac{2T \times EF}{1 + CIR} \frac{CIR}{\eta + CIR(1-\eta)} = \frac{2}{1 + CIR} \frac{CIR/\eta}{1 + CIR\frac{1-\eta}{\eta}} \frac{SE}{V_1 I_1} \tag{59}$$

This damping time constant is independent of the switching frequency f. It can be used to estimate the oscillation responses for step function and impulse interference. The ratio ξ is

$$\xi = \frac{\tau_d}{\tau} = \frac{CIR}{\eta(1 + CIR\frac{1-\eta}{\eta})^2}. \tag{60}$$

6 Transfer function of power DC/DC converters

A DC/DC converter usually has two or more energy-storage elements. The *time constant* τ and *damping time constant* τ_d are used to form the transfer function of a power DC/DC converter describing its characteristics with a second-order differential operation for a small signal analysis. The voltage transfer gain of the DC/DC converter is $M = V_2/V_1$. The transfer function of the DC/DC converter can be modelled as

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{M}{1 + s\tau + \xi s^2\tau^2} \tag{61}$$

where M is the voltage transfer gain V_2/V_1, τ the time constant (58), τ_d the damping time constant (59), $\tau_d = \xi\tau$.

Using this mathematical model of power DC/DC converters, it is easy enough to describe the characteristics of power DC/DC converters. In order to verify this theory, we will use two converters to demonstrate the characteristics of power DC/DC converters and applications of the theory.

6.1 Buck converter

Fig. 5 (a) shows a Buck converter with the conduction duty k [9,10]. The components values are the same as in Sect. 3.1 and there are some power losses, described by the inductor resistance $r_L = 4\ \Omega$. We then obtain $V_2 = 11.4$ V, $I_2 = I_L = 1.14$ A, $P_{loss} = I_L^2 r_L = 1.14^2 \times 4 = 5.2W$, $I_1 = 0.455$ A, which gives

$$PE = V_1 I_1 T = 0.91 \text{ mJ}, \qquad W_L = \tfrac{1}{2} L I_L^2 = 0.162 \text{ mJ}$$

$$W_C = \tfrac{1}{2} C V_C^2 = 3.9 \text{ mJ}, \qquad SE = W_L + W_C = 4.06 \text{ mJ},$$

$$EF = \tfrac{SE}{PE} = \tfrac{4.06}{0.91} = 4.463, \qquad CIR = \tfrac{3.9}{0.162} = 24$$

$$EL = P_{loss} * T = 5.2 * 50 = 0.26 \text{ mJ}, \qquad \eta = \tfrac{PE-EL}{PE} = 0.714$$

$$\tau = \tfrac{2T \times EF}{1+CIR}(1 + CIR\tfrac{1-\eta}{\eta}) = 189.3\mu\text{s}, \qquad \tau_d = \tfrac{2T \times EF}{1+CIR}\tfrac{CIR}{\eta + CIR(1-\eta)} = 56.6\ \mu\text{s}.$$

Since $EF > EF_V$, this converter is stable. The power transfer efficiency $\eta = P_O/P_{in} = 13/18.2 = 71.4\%$. Since $\xi = \tau_d/\tau = 0.299 > 0.25$, the transfer function of this Buck converter has two poles ($-s_1$ and $-s_2$) that are located in the left-hand half plane (LHHP):

$$G(s) = \frac{M}{1 + s\tau + s^2 \tau\tau_d} = \frac{pk/\tau\tau_d}{(s+s_1)(s+s_2)} \tag{62}$$

where

$$s_1 = \sigma + j\omega \text{ and } s_2 = \sigma - j\omega$$

with

$$\sigma = \frac{1}{2\tau_d} = \frac{1}{113.2\mu s} = 8.83 \text{ kHz} \tag{63}$$

and

$$\omega = \frac{\sqrt{4\tau\tau_d - \tau^2}}{2\tau\tau_d} = \frac{\sqrt{42857.5 - 35834.5}}{21428.76} = \frac{73.8}{21428.76\mu} = 3.911 \text{ krad/s},$$

$$p = 0.714M = pk = 0.285. \tag{64}$$

The step function response in the time-domain is

$$V_2(t) = 11.4[1 - e^{-\frac{t}{0.000113}}(\cos 3911t - 2.26 \sin 3911t)] \text{ V}. \tag{65}$$

The step function response (transient process) has an oscillation character with the damping factor σ and frequency ω and is shown in Fig. 14.

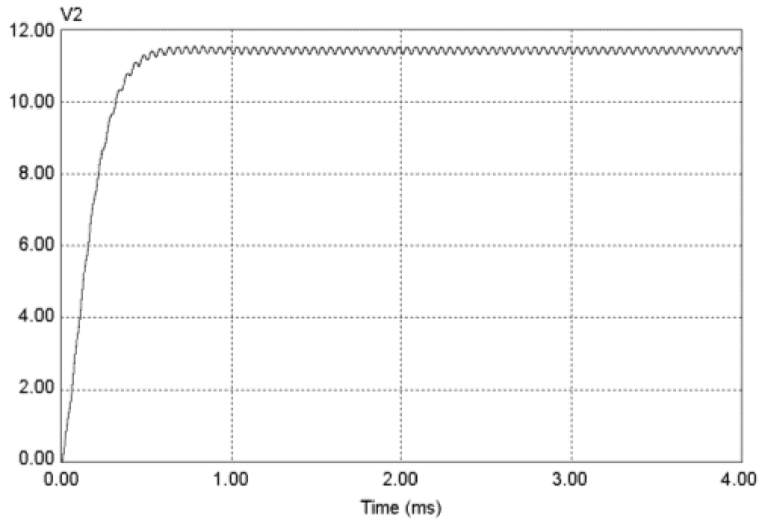


Figure 14: Step function response of a Buck converter with $r_L = 4\ \Omega$.

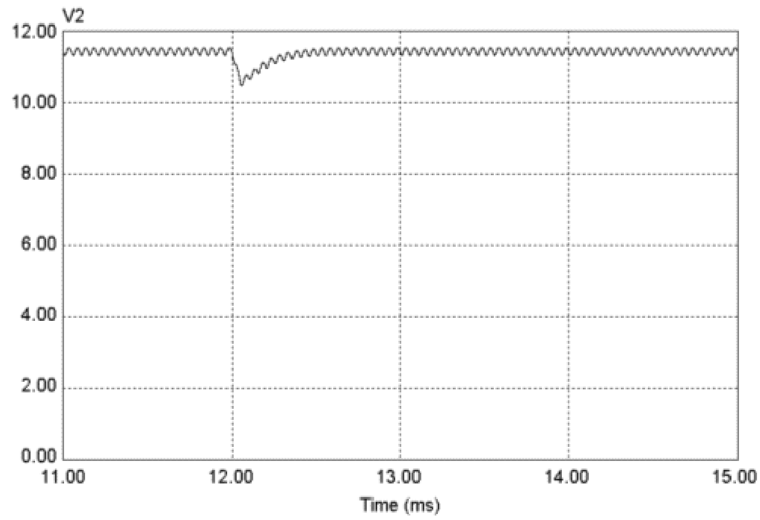


Figure 15: Impulse response of a Buck converter with $r_L = 4\ \Omega$.

The impulse interference response in the time-domain is

$$\Delta V_2(t) = 4.52 U e^{-\frac{t}{0.000113}} \sin 3911t \tag{66}$$

where U is the interference signal. The impulse response (interference recovery process) has an oscillation character with the damping factor σ and frequency ω, and is shown in Fig. 15.

6.2 Super-Lift Luo-Converter

Fig. 12 shows a Super-Lift Luo-Converter with the conduction duty k [9,10,14-18]. The components values are $V_1 = 20$ V, $f = 50$ kHz ($T = 20\mu$s), $L = 100\mu$H, $k = 0.5$, $C_1 = 2500\mu$F, $C_2 = 800\mu$F, and $R = 10\ \Omega$. There are some power losses described by the inductor resistance $r_L = 0.12\ \Omega$. We then obtain $V_2 = 57.25$ V, $I_2 = 5.725$ A, $I_1 = 17.175$ A, $I_L = 11.45$ A, $P_{loss} = I_L^2 \times r_L = 11.45^2 \times 0.12 = 15.73$ W, $V_{C1} = V_1 = 20$ V, $V_{C2} = V_2 = 57.25$ V. It is operating in CCM, the parameters are

$$\begin{aligned}
&PE = V_1 I_1 T = 20 \times 17.175 \times 20\mu = 6.87\text{mJ},\\
&W_L = \tfrac{1}{2} L I_L^2 = 0.5 \times 100\mu \times 11.45^2 = 6.555 \text{ mJ},\\
&W_{C1} = \tfrac{1}{2} C_1 V_{C1}^2 = 0.5 \times 2500\mu \times 20^2 = 500\text{mJ},\\
&W_{C2} = \tfrac{1}{2} C_2 V_{C2}^2 = 0.5 \times 800\mu \times 57.25^2 = 1311 \text{ mJ},\\
&SE = W_L + W_{C1} + W_{C2} = 1817.6 \text{ mJ},\\
&EF = \frac{SE}{PE} = \frac{1817.6}{6.87} = 264.6\\
&CIR = \frac{1811}{6.555} = 276.3\\
&EL = P_{loss} * T = 15.73 * 20 = 0.3146 \text{ mJ},\\
&\eta = \frac{PE - EL}{PE} = 0.9542\\
&\tau = \frac{2T \times EF}{1 + CIR}\left(1 + CIR\frac{1-\eta}{\eta}\right) = 38.168 \times 14.26 = 544.35\ \mu\text{s},\\
&\tau_d = \frac{2T \times EF}{1 + CIR}\frac{CIR}{\eta + CIR(1-\eta)} = 38.168 \times 20.3 = 774.93\ \mu\text{s}.
\end{aligned}$$

Since EF > EF_V, this converter is stable. Its time constant $\tau = 0.544$ ms and damping time constant $\tau_d = 0.775$ ms $= 1.42\tau$ ($\xi = 1.42$). The transfer function of this converter has two poles ($-s_1$ and $-s_2$) that are located in the left-hand half plane (LHHP):

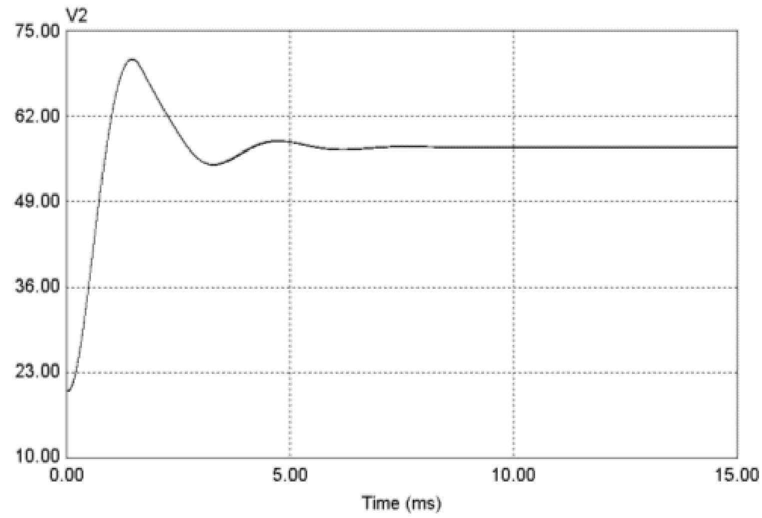


Figure 16: Step function responses of Super-Lift Luo-Converter with $r_L = 0.12\ \Omega$.

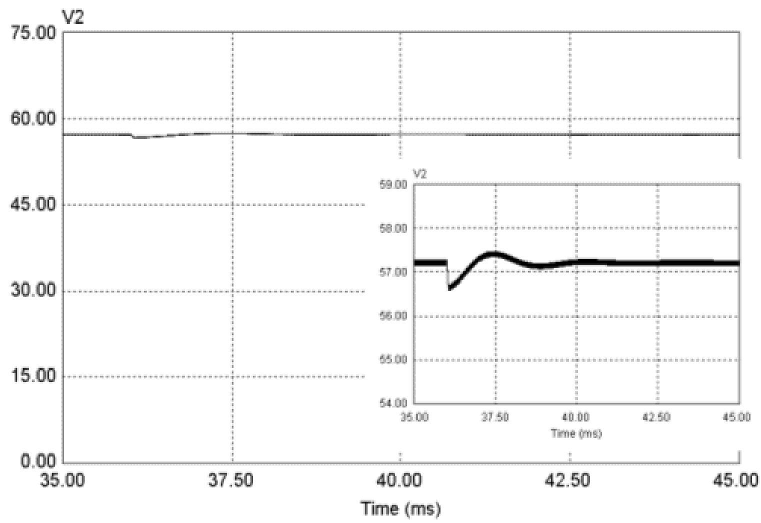


Figure 17: Impulse responses of Super-Lift Luo-Converter with $r_L = 0.12\ \Omega$.

$$G(s) = \frac{M}{1 + s\tau + s^2\tau\tau_d} = \frac{M/\tau\tau_d}{(s+s_1)(s+s_2)} \tag{67}$$

where

$$s_1 = \sigma + j\omega \text{ and } s_2 = \sigma - j\omega$$

with

$$\sigma = \frac{1}{2\tau_d} = \frac{1}{1.55s} = 0.645 \text{ Hz} \tag{68}$$

and

$$\omega = \frac{\sqrt{4\tau\tau_d - \tau^2}}{2\tau\tau_d} = \frac{\sqrt{1686400 - 295936}}{843200} = \frac{1197.2}{843200\mu} = 1.3985 \text{ krad/s}, \tag{69}$$

$$\frac{1}{\sqrt{4\tau_d/\tau - 1}} = \frac{1}{\sqrt{5.69853 - 1}} = \frac{1}{2.1676} = 0.461,$$

$$M = 3x0.9542 = 2.8626.$$

The step function response in the time-domain is

$$V_2(t) = 57.25[1 - e^{-\frac{t}{1.55}}(\cos 1398t - 0.461 \sin 1398t)] \text{ V} \tag{70}$$

The step function response (transient process) has an oscillation character with the damping factor σ and frequency ω and is shown in Fig. 16.

The impulse interference response in the time-domain is

$$\Delta V_2(t) = 0.923Ue^{-\frac{t}{1.55}} \sin 1398t \tag{71}$$

where U is the interference signal. The impulse response (interference recovery process) has an oscillation character with the damping factor σ and frequency ω, and is shown in Fig. 17.

7 Experimental results for power DC/DC converters

To verify the analysis, calculation, and simulation given in the previous sections, we constructed the test rig to complete some experiments. The results are listed in the following subsections.

7.1 Buck converter

The circuit diagram corresponds to Fig. 5 (a), and the components values are same to those in Sect. 3.2. There are some power losses described by the inductor resistance $r_L = 1.5\ \Omega$. We then obtain the experimental resultsfor the step response and impulse response shown in Figs. 18 and 19. We can find out that the experimental results are identical to the simulation results in Figs. 8 and 9.

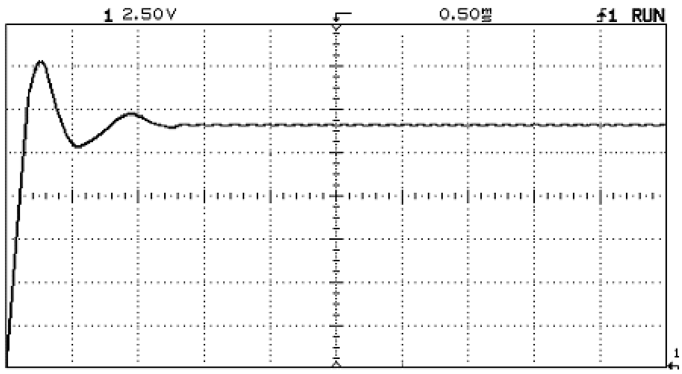


Figure 18: Step function response of a Buck converter with $r_L = 1.5\ \Omega$ (experiment).

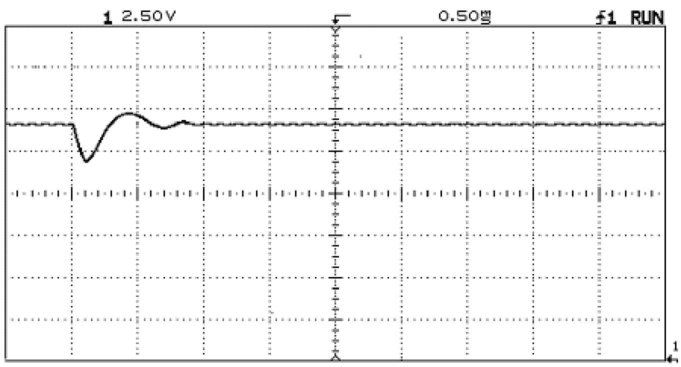


Figure 19: Impulse response of a Buck converter with $r_L = 1.5\ \Omega$ (experiment).

7.2 Super-Lift Luo-Converter

The circuit diagram corresponds to Fig. 12, and the components values are same to those in Sect. 6.2. There are some power losses described by the inductor resistance $r_L = 0.12\ \Omega$. We then obtain the experimental results for the step response and impulse response shown in Figs. 20 and 21. We can find out that the experimental results are identical to the simulation results in Figs. 16 and 17.

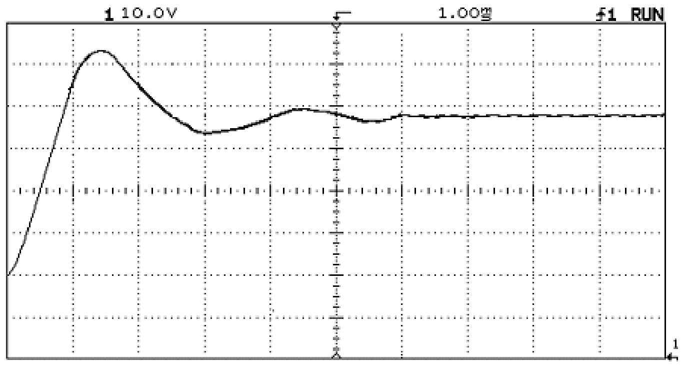


Figure 20: Step function responses of Super-Lift Luo-Converter with $r_L = 0.12\ \Omega$ (experiment).

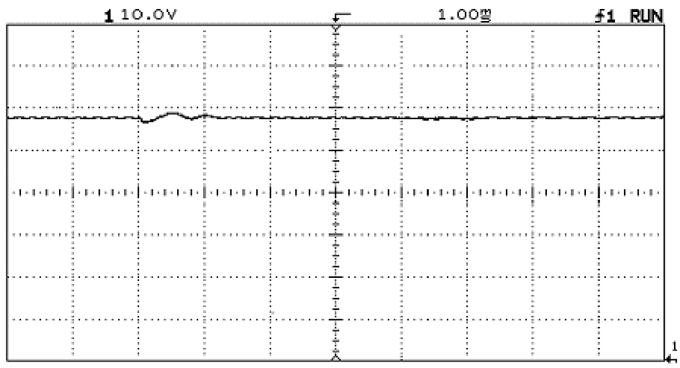


Figure 21: Impulse responses of Super-Lift Luo-Converter with $r_L = 0.12\ \Omega$ (experiment).

8 Conclusion

Mathematical modeling of power DC/DC converters is a historical problem. The traditional mathematical modelling is not suitable for complex structure converters due to a dramatic increase in the order of a corresponding differential equations. We have to search other way to establish mathematical modelling for power DC/DC converters. This paper offers a mathematical model (71) generally adequate for power DC/DC converters. Their parameters are determined by completely new concepts: *Energy Factor* (EF) and subsequential parameters.

Since traditional parameters such as efficiency (η), power factor (PF), THD, and RF cannot present the characteristics of power DC/DC converters, the authors are the pioneers to define Energy Factor (EF) and other parameters to describe characteristics of power DC/DC converters. Using these parameters allows one to demonstrate all characteristics of power DC/DC converters. Two typical converters, Buck converter and Super-Lift Luo-Converter, are employed to perform these parameters, and satisfactory simulation and experimental results are obtained. It means that the *Energy Factor* (EF) and other parameters are very helpful in Power Electronics and DC/DC conversion technology.

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