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MATHEMATICAL METHODS AND MODELS IN ENVIRONMENTAL STUDIES

Environmental quality and satisficing games

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Abstract

This paper outlines selected facets of environmental quality and strategic risks and control. In particular, an environmental games approach is emphasized and a "satisficing" solution is suggested. Such a concept is motivated by the complexity of environmental games, the partial information available and by a claim that in such circumstances, the traditional Nash equilibrium might not be appropriate. A specific application - an environmental game, is used to demonstrate the approach we follow and draw some conclusions regarding the process of investment in technologies for policy abatement and environmental controls.

1 Introduction

Current conventional wisdom, trumpeted at yearly NGO meetings and on many political platforms, states a common message: for a society to be sustainable it has to maintain the quality of its environment. As a result, the environment quality has become a strategic topic for those concerned about the environment (for example, [1–4]). To "produce environmental quality", regulators and environmental managers in potentially polluting firms must become aware of the mutual relationships and inter-dependencies of investments in pollution abatement technologies, in the control effort they exercise on the processes under their control and the control regulators can exercise. Both economic and environmental quality considerations are to be addressed. And, at the same time, resolve the inner societal conflicts that environmental protection and quality involve. For example, protecting fish or animals and facing a starving population may not be realistic. By the same token, controlling gas emissions of cars while penalizing public transportation may also have counter effects on the quality of the environment.

The measurement and the definition of environmental quality are intimately related. For example, many consumer groups support proposals to assure some measure of environmental quality and accountability. But some experts believe that this is of limited value because of problems associated to gathering data on which consumers can agree or collect data which are statistically meaningful. Further, in some cases, conflicting interests and a weak underlying scientific data base are likely to lead to court litigation rather than to a resolution of environmental problems. Subjective benchmarks like a popular satisfaction index can be misleading, leading to plans and resources to score high marks.

Environmental quality management and control thus involves people and their widely differing points of view, economic and technological problems, regulation, application of audits, scientific know-how, uncertainties, incomplete and partial information, engineering design and a complex (often involving conflict) framework that needs at the same time to integrate, to monitor, and to control the multiple elements that underlie the quality of the environment (for some references see also [5, 6]).

Environmental quality is thus, necessarily, pluri-disciplinary, involving simultaneously the many facets of environmental quality and its control. By the same token, the analysis and policy recommendations of problems associated to environmental quality and control must use concepts and systems that recognize this complex reality. The purpose of this paper is to outline selected facets of environmental quality, its definition, the concept of environmental risks controls—emphasizing risk and conflict and at the same time provide some elementary approaches to deal with environmental problems that recognize these facets of the environment. In particular, an environmental games approach shall be emphasized and a "satisficing" solution concept will be suggested. Such a concept is motivated by the complexity of environmental games, the partial information available and therefore by my claim that in such circumstances, the traditional Nash equilibrium might not be appropriate. A specific application—an environmental game is used to demonstrate the approach we follow and draw some conclusions regarding the process of investment in technologies for policy abatement and environmental controls.

2 Environmental risks and quality

Environmental quality: definitions

A definition of environmental quality is important for it may mean different things in various circumstances and to several actors, each responding to its specific needs. ISO for example, defines quality as the totality of features and characteristics of a product or service that bear on its ability to satisfy stated or implied needs. In contrast, environmental quality can have several attributes that have various meanings—potentially contradictory, to several, agreeing or disagreeing groups on the measurements applied to specify quality. There may also be objective—measured based and subjective attributes, expressing both tangible and intangible characteristics of environmental quality (e.g. see [7]). For example, two essential approaches to environmental quality and quality in general including: Quality Assurance and Quality Improvement, have multiple purposes eyeing environmental polluters, regulators, environmental sensitive NGO's and the society at large. Some of these purposes in case of pollution management include, for example:

- the detection of unacceptable emission by polluters with the intention of preventing them and maintaining an acceptable level of environmental quality;
- to identify potential and unacceptable behavior of potential polluters with the intention of improving the quality of the environment;
- to increase the efforts expanded by potential polluters—both in pollution abatement investments and in preventive controls and to prevent the degradation of existing acceptable levels of care by firms and finally
- to motivate potential polluters to ever higher levels of care and controls. Further, due to the complexity of environmental control technologies and the valuation of required efforts, inherent information

asymmetries and difficulties in setting a price for environmental quality, it is important to emphasize the need for Continuous Improvement which has been successfully applied in industry and can be applied as well for environmental protection.

While these bullet points may seem obvious to some, they are likely to raise as well heated discussions when confronted with the multiple realities of environmental quality management. In this sense, it may be useful to expand on the basic premises that underlie environmental quality, how to define it, how to measure it, and what are its implications and risks [8].

"Zero pollution" for example, is based on the Olympic concept, "Citius, Altius, Fortius" meaning "Faster, Higher, Stronger" engraved on Olympic medals and symbolizing the relentless pursuit of an ever greater excellence in man's environment. Quality of life and quality of the environment are then an expression of this excellence and to society's claims to its quality of life. In this context, a society is perpetually challenged to improve and control the environment and the many participants oblivious to its quality (firms, polluters, the population at large, in the present and in the future). Alternatively, the "Quality of Life and Environmental Quality" may be conceived as relative concepts! Is the concentration of carbon dioxide increasing or decreasing? Are persons happier? Such questions, may in some cases be dealt with a seemingly sense of objectivity. In other words, defined by implication in terms of attributes and scales used to measure and combine these attributes. In some cases, these attributes may be observed and measured precisely. However, they can also be difficult to observe directly and impossible to measure with precision. These situations are some of the ingredients that make environmental quality the intangible variable society has difficulties dealing with. Nevertheless, a combination of such attributes, in "various proportions" can lead to the definition of environmental quality. In this sense, quality is defined relative to available alternatives and can be measured and valued by some imputation associated to these alternatives. There is no agreement how to proceed in such measurements however. "Quality is not what we think but what the society says environmental quality is"—underlies the "consumerism" and modern approach to measurement of quality in general, emphasizing a downstream (and democratic) orientation. Industrialists often defend themselves in polluting by claiming that environmental quality is what consumers are willing to pay for in more expensive products and services. Such views are, of course, motivated by the need to value environmental quality so that sensible decisions regarding a firm's investment in pollution abatement, for example, can be reached. Namely, how much would society be willing to pay for environmental quality? The rising trend and hype regarding sustainable development is particularly important in this matter for it increases firms' awareness that the payoffs and costs of an environmental unquality can be substantial and lingering over the long run, thereby motivating them to assume in the present some preventive measures. For example, how much is an individual willing to pay for clean air if it also implies that he would not be free to use his car in the city for two days a week? This is, of course, measured by the value added of clean air versus the value added of being able to drive one's own car freely. At the same time, it is quite evident that with no emission controls, society's costs will not be sustainable with car manufacturers facing a sudden demise in case no preventive actions are taken in the present. There are already competitive effects today to these problems with forthcoming car emissions regulations which promises to render car manufacturing more expensive (over 450 dollars for Ford and GM cars while under 80 dollars for Japanese cars who have invested premptively in more efficient gas emission engines).

As a result, environmental quality is not a term which can be defined simply. Rather it is a composite term, expressed in terms of attributes which define quality by implication. These attributes express:

- *The definition and relative desirability of environmental quality attributes*
- *Substitution, differentiation and sustainability, both objective and subjective, and, of course, over time in valuing the present versus the future*
- *The parties involved in the environment, both public and individual*
- *The uncertainty that besets the knowledge, the definition and the measurement of environmental quality.*

If environmental attributes are not substitutes (meaning that they are not comparable and one cannot measure one with respect to the other), then environmental quality as a variable used to compare environmental states is not relevant. Differentiation can be subjective, perceived differently by the many parties concerned by the environment. Smell, lighting, clean air are perceived differently. If persons "were the same" in terms of how they value and assess characteristics associated with the environment, then they may be considered homogeneous and the concept of environmental quality would be well defined in terms of agreed on properties.

Risks and environmental quality

Risk (unlike environmental quality) results from the direct and indirect adverse consequences of outcomes and events that were not accounted for or that we were ill prepared for, and concerns their effects on individuals, firms or the society at large. It can result from many reasons, internally induced or occurring externally. In the former case, consequences are the result of failures or misjudgments while in the latter, consequences are the results of uncontrollable events or events we cannot prevent. A definition of risk involves as a result four factors: (i) consequences (ii) their probabilities and their distribution (iii) individual preferences, and (iv) collective and sharing effects [9, 10]. These are relevant to a broad number of fields, and not only to environmental problems, each providing a different approach to the measurement, the valuation and the management of risk which *is motivated by a need to deal with problems that result from uncertainty and the adverse consequences they may induce.* For these reasons, the problems of risks and their management are applicable to many fields, each bringing its own experience, wisdom and tooling that together, may contribute to managing the quality of the environment. Financial economics, for example, deals extensively with the pricing of risks and hedging which is very important to assess in fact what is the cost of environment related investments in pollution abatement, in imposing certain environmental regulation, in determining the premium to be paid for and shared for environmental insurance and their like [9, 10]. Industrial management has also contributed largely to managing "unquality", measuring and detecting (controlling) deviations from "agreed or specified standards of performance" [8, 11]. Each discipline devises the tools it can apply to minimize the more important risks it is subjected to. In environmental management, for example, risks may pertain to the probabilities of polluting

events and their consequences (minor pollution to large disasters) occurring with regular and large probabilities or with very small probabilities. Risk consequentially affects environmental quality. However, they need not mean the same thing nor be treated in the same manner. In environmental management, risks may have an effect which is not sustained by the responsible party (for example, a polluting firm might not be the sole victim and pollution effects may be felt over long periods of time by the population). To manage environmental quality, however, we cannot negate the importance of environmental risks. Questions such as: who pays for it; what prevention if at all; who is responsible if at all, etc. are essential problems we must address if we are to begin to manage the quality of our life and our environment. In this quest, problems of information asymmetries, inducing moral hazard, conflicting objectives, long and short term considerations can lead to behaviors which can have detrimental effects on environmental quality. In such circumstances, negotiations as well as monitoring and controlling contract conformance become a strategic facet of the management of environmental quality. For example, incentives and controls are then needed to assure that potential polluters perform as intended. Controls assume therefore a dimension far broader and far more important than applied in very limited industrial settings.

Obviously, if the cost of environmental pollution is well defined, the measure of that value is what makes it possible to distinguish between various qualities. When costs are uncertain or intangible, their measurements are more difficult and therefore environmental quality is harder to express. In this sense, uncertainty has an important effect on the definition, the measurement, the risks and the management of quality in general and in the environmental quality in particular.

3 Environmental quality, uncertainty and conflict

Statistics and control have traditionally been concerned with the control of uncertainty, seeking to monitor it, to predict it, limit its effects, and whenever possible to control it. Quality control, stochastic control, and general decision making under uncertainty are some of the fields which are involved, in one way or the other, in an attempt to deal with these problems which have plagued our profession whenever it has been confronted with uncertainty.

At the same time and often in an unrelated manner, a theory of decision making under conflict has been devised. Foremost is the definition of solutions, based on equilibria, such as the well known Nash equilibrium of game theory [12–14]. The contribution of the Israeli school to these problems is particularly noteworthy. This has grown into an important field of study and numerous applications in Economics, Environmental and Management Science have been devised. There are too many applications to be elaborated here.

The relationship between statistics, conflict and control, as well as the role of statistical sampling in improving the control of conflict, have to a large measure been neglected. Inversely, the importance of conflict and gaming to the design of statistical sampling has also been a topic largely neglected. Statistic's failure to deal with conflict arose from its presumption that "uncertainty is not motivated". In other words, randomness is an act of G-D which has no known or is not directed towards any special purpose. Interpreting uncertainty and reducing its effects is then based on the presumption that our measurements and our acts are independent of the origins of such uncertainty. On the other hand, game theory has for the most part neglected uncertainty (or equivalently, has reduced it) and dealt only with the uncertainty in the conflict which underlies the decision making environment of two or more parties. Special problems have been pointed out which require special attention; "moral hazard and adverse selection" as stated earlier. These problems arise as a consequence of information asymmetry between parties of conflicting interest that induces a greater need for controls, to assure that "what is intended will occur". Here again, the use of sampling as a technique to mitigate the effects of information asymmetry on decision efficiency in a conflictual environment have been ignored. For example, strategic environmental audits have always a number of messages they convey; a control, a signal to the audited and of course to collect information which is needed to reach an economic decision. For example, in a bilateral monopoly (i.e. when only two parties are involved in a decision making, information asymmetry can lead to opportunistic behaviour, or simply said–cheating). The control of exchanges between such parties should therefore keep in mind parties' intentionalities imbedded in their preferences, the exchange terms as well as the information each will use in respecting or not the intended terms of their exchange. In some papers, Reyniers and myself [15, 16] have applied such an approach to the design of ex-post contract monitoring when the control of quality occurs between parties with various motivations. More recently, I have also dealt with environmental quality games [17–19]. In the next section an application to this effect will be considered. Unfortunately, for such games the concept of Nash (and its variants) might be difficult to apply (and in my personal view, perhaps providing a solution which is too conservative). Rather, I shall concentrate some attention on a solution concept for environmental games I shall coin "satisficing". The motivation for such a solution concept arose from both computational difficulties met in the solution of environmental games that have random outcomes and partial information available to each of the environmental "players". In this case, the essential properties required for an environment game-like solution based on stability (equilibrium), efficiency, and fairness might be difficult to find and might not exist.

Satisficing and Laplace's principle of insufficient reason

The concept "satisficing solution" to a game is based on two essential presumptions. First, that game participants have objectives they want to meet (constraints) absolutely or in a probabilistic manner, by specifying the risks they are willing to sustain and associated to each potential strategy. Second, it presumes that in the absence of a well defined objective (except, of course, those specified through an appropriate set of constraints), a solution is defined by strategies that assume the least regarding players motivation. In other words, it applies the principle of Laplace of Insufficient Reason to the game solution. In this sense, we do not consider a Nash like solution to the environmental game but a less conservative "satisficing" solution. Thus, we do not consider environmental games as mathematical games of pure conflict but rather games with nuances of potential collaboration. A comparison between Nash solutions and "satisficing" solutions in any particular game provides as a result, a potential of "least collaboration", meaning that acting in a manner that the opposing agent will not seek to act purposefully as players in conflict. Of course, such a solution will be less than one could obtain by a cooperative solution, but then such solutions may not be easily enforceable.

For practical purposes and in order to calculate a "satisficing" random strategy solution for each of the players we may apply a Maximum Relative Entropy objective, for both agents. Such an approach was used by Neyman and Okada [20] in a context of repeated games, providing an iterative approach which is information sensitive. Kapur and Kesavan [21, p.297] have remarked as well that every probability distribution, theoretical or observed, is an entropy optimization distribution, i.e., it can be obtained by maximizing an appropriate entropy measure or by minimizing a cross-entropy measure with respect to an appropriate a priori distribution, subject to its satisfying appropriate constraints (see also [22]). In this sense, once a game with an appropriate set of constraints is specified, it is possible to apply the Principle of Maximum Entropy to determine the "satisficing" randomized strategies of the game (and thereby the statistical controls applied in managing the games' strategies). An example to this effect is considered in the next section.

Computationally, *Maximizing Entropy* subject to a given set of constraints allows selecting that distribution which assumes the "least"–that is the distribution with the greatest variability, given the information and the available constraints. Its use is justified and based on the notion that the distribution with the largest randomness is also of maximal entropy. Its origins arose in statistical physics. Boltzmann observed that entropy relates to “missing information” inasmuch as it pertains to the number of alternatives which remain possible to a physical system after all the macrospically observable information concerning it has been recorded. In this sense, we can interpret “information” as that which changes a system’s state of randomness (or equivalently, as that quantity which reduces entropy). The use of a Maximum Entropy principle is justified further by its convenience, for it can be applied relative to some other distribution presuming a given knowledge regarding the agents participating in the environmental game. For this reason, we coined application of this rule as a Maximum Relative Entropy.

For example, say that an opponent’s randomized strategy is presumed to have a probability distribution G when in fact it is a distribution F. The "degree of certainty” for this assertion will be denoted by U(G/F). Similarly, we may believe that the opponent’s strategy is affected by a number of random elements due to random payoffs, external events and their likes (say, a random variable x) while in fact it is affected by something else (say, a random variable y). We denote the degree of this assertion by the measure of certainty v(x/y). The problem at hand is how to characterize quantitatively these assertions. I.J. Good [23] has shown that under the following fours assumptions-axioms, a “measure of certainty” can be uniquely defined (see also [24, 25]). These axioms are:

- If a constant is added to v, then the same amount is added to U
- If we have two mutually dependent random variables x and x* with probability densities F and F* and G and G* respectively, then the degree of certainty of the joint distributions equals the sum of the “certainty measures", or U(G,G*/F,F*)=U(G/F)+U(G*/F*)
- Invariance under non singular transformation of the random variable x, U(F/F)>=U(G/F) for all F and G and also v(x,y)>=v(x,y) for all x and y.

In particular, if the presumed distribution of G is g, then:

$$U(G\,|F\,) = \int \log\left\{g(x)\,|\Delta x|^{-1/2}\right\} dF(x)$$

where

$$\Delta x = \left\{\frac{\partial^2 v(x,y)}{\partial y_j \partial y_k}\right\}_{y=x} ; j,k = 1,2.$$

If F has a probability density f(x), dF(x)=f(x)dx and therefore, the certainty of asserting that $x \in F$ when in fact it belongs to F is given by:

$$U(F\,|F) = \int f(x) \log \left\{ f(x) \, |\Delta x|^{-1/2} \right\} dx.$$

Hence, given Δx, minimizing the functional U(F/F) will be equivalent to choosing the distribution that assumes the least. Jaynes has called this, the principle of invariantized entropy [23]. This principle generalizes in some ways the principle of insufficient reason which is pointed out to by Laplace and by Bayes. The choice of Δx is inherent in the characterization of states of complete ignorance. When Δx=1, we obtain the maximum entropy principle. In Jaynes, $|\Delta x|^{1/2} = m(x)$ is an "invariant measure" function, making the entropy invariant under transformation of variables and proportional to the limiting density of the discrete points of the distribution. Good [23] refers to the term Δx as the Fisher information matrix.

Say that we have no information and take an interval of length $x \in [a, b]$. Then, minimization of U will yield the equiprobable distribution:

$$f(x) = \frac{m(x)}{\int_a^b m(x)dx}; a \le x \le b.$$

As additional information is acquired, constraining the optimization process, another distribution will be obtained. This principle implies that when a player is subject to some information inputs, he then chooses among the families of distributions containing his known information and which is consistent with the presumed rationality and information of his opponent. This behavior constitutes a conservative strategic choice. In this regard, we may claim that the player does not infer more than given by the information and the rules of the game. Some of these concepts have been applied to game theory. In some cases, prior knowledge may point out to probabilities currently practiced. This is the case in repeated games where at each game run, each of the players has a probabilistic assessment of the other player which can, of course, be updated after each game run [20]. Similarly, Stirling [26] and Stirling and Goodrich [27] have developed an axiomatic foundation for determining a "satisficing" solution to games based on comparative solutions which are far more practical when applied to multi-agent problems. They claim as well that "satisficing solutions" are not meant to discredit the Nash approach but rather to complement it when dealing with problems where the Nash solution cannot be implemented effectively. Similarly, Bendor and Kumar [28] conclude in a Working paper on "Satisficing and Optimality", that "what's bad news normatively may be good news methodologically...". In particular, they formalize notions of satisficing and search, and show that in a wide array of contexts a satisficing agent will not converge to optimizing. Further, they show that in a broad range of environments, a pair of adaptively rational players will not be absorbed into Nash outcomes of the stage game. They explain such behavior by showing that satisficing-and-search rules genuinely differ from optimization (or Nash behavior) even in the long-run and even in stationary environments. The root cause of this difference is that these adaptive rules do not optimally trade-off exploration versus persistence. Finding an optimal solution usually requires a willingness to explore new options, but keeping an acceptable one may require a wise persistence in the face of disappointment. In an environmental setting, often ill defined and with many parties, "satisficing" solutions may, perhaps, not provide the "best" or normative solution we may be looking for, but it may lead to a workable solution. In the next section, an application is considered by solving a game in its traditional Nash solution sense and in applying the concept of a satisficing solution.

4 An application

The environmental game we consider and to which we shall apply the concept of "satisficing solution" is defined between two parties: an environmental "regulator" and a potentially "polluting firm". We assume that the firm uses a pollution technology, expressed by the propensity to pollute and with a "pollution production" cost function given in terms of the propensity to pollute. The smaller this probability, the larger the "production" cost. Pollution risks, measured by their consequences are, however, functions of the regulators controls. A polluting event which is not detected, is costless to the firm but costly to "society" faced with cleaning the environment. A polluting event which is detected, induces a cost borne by both the firm and "society". Costs are assumed shared in some fashion according to an agreed on sharing rule. The firm can invest in preventive measures, such as controlling its processes, while an environmental regulator can invest and augment controls. In the former case, we assume that no pollution damage can be made while in the latter, damages, once detected, incur penalties that the firm must sustain. The resulting environmental game is used to draw some conclusions regarding the process of investment in pollution abatement technologies and the preventive efforts and controls to exercise by polluting firms while at the same time, determining the control effort that environmental regulators ought to apply in both Nash type solutions and "satisficing" solution.

The problems that both the firm and the regulator are faced with are then two-fold: (1) given a polluting technology and a shared penalty cost for polluting events, what are the control effort to exercise by the firm and what control and preventive efforts to exercise and (2) what are the effects of the technology choice and penalty-cost sharing parameters on the firm and society's payoffs (see also [17]).

In order to focus the paper's attention, we consider an active firm, potentially polluting by emiting a noxious gas, for example, by shipping oil in tankers with various probabilities of creating oil spills, etc. In particular, we assume that a firm uses a pollution technology (resulting from an investment in a pollution abatement technology), and pollutes with probability p. In addition, the firm can invest in preventive controls whose cost is c_F. For example, assume an industrial polluter (through an uncontrolled emission) with probability p. If preventive efforts are invested at a cost $C(p)$ then no pollution occurs. A firm may thus select both the pollution technology and the preventive efforts it applies. Production costs $C\left(p\right)$, are assumed to be a function of the polluting probability p with $\partial C/\partial p < 0$, $\partial^2 C/\partial p^2 < 0$, $C(0) = \infty$. In this sense, a firm's pollution strategy is defined by selecting the probabilities (p, x) of polluting and of exercising a preventive effort whose costs is c_F. By the same token, an environmental regulator has the choice to control (with probability y) or not a "work event" of the firm, which may result or not in a polluting event. In case such an event occurs and is detected, then a penalty cost might be applied to the firm. Since there are "no winners in pollution", both the firm and society at large end up penalized by such an event. These situations result in a random payoff game given in Table 1 below.

Note that for the firm, the basic payoff from its activity, as long as it is not detected, is $\pi_P - C(p)$ while the payoff if it applies preventive controls is $\pi_P - [C(p) + c_F]$. When the firm does not apply preventive measures and a polluting event is detected by the regulator, then the firm payoff is random, with $(1-\alpha)\tilde{\Phi}$ an additional penalty cost is sustained where $\tilde{\Phi}$ is the damage resulting from the polluting event (a random variable) and $(1-\alpha)$ is the firm share. The complementary share is sustained by society. Next, let π_S be society's payoff that arise from the activity of the firm while c_S is the cost of applying an environmental control. These situations summarize the costs of by the firm and society and are given in Table 1.

An environmental regulator's strategy consists then in either controlling-

	Control by Regulator y	No controls by Regulator $1-y$
Control by Firm x	$\pi_P - [C(p) + c_F]$ $\pi_S - c_S$	$\pi_P - [C(p) + c_F]$ π_S
No Control by Firm $1-x$	$\begin{cases} \pi_P - C(p) - (1-\alpha)\tilde{\Phi} & p \\ \pi_P - C(p) & 1-p \end{cases}$ $\begin{cases} \pi_S - c_S - \alpha\tilde{\Phi} & p \\ \pi_S - c_S & 1-p \end{cases}$	$\pi_P - C(p)$ $\begin{cases} \pi_S & 1-p \\ \pi_S - \tilde{\Phi} & p \end{cases}$

Table 1: The (Polluter, Regulator) Payoff Matrix

auditing the polluter or not while the firm strategy is two-fold. Should it resort to preventive controls or not and what would be the level of its polluting technology. These result in a random payoff two persons non-zero sum game (see also [8, 15, 16, 29] for related studies). Randomness occurs therefore from two sources, the mutual controls set by the polluting firm and the regulator who acts as a proxy for society and, of course, randomness of the pollution events (p) and finally randomness of the cost of these events ($\tilde{\Phi}$).

Uncertainty in environmental problems primes therefore at several levels and is determined as well by the acts that each of the parties, bound by risk sharing and legal agreements and conflicting objectives for the firm and society, will apply. Environmental controls and their costs combined with environmental pollution, abatement technology, and detection technologies are thus required and determine the participants' behavior and propensity to pollute. In other words, environmental controls are determined by the solution of a game, which recognizes the realistic conflict between the polluter and the regulator and the uncertainty such conflict generates. Finally, we clarify the idea of equilibrium as a mechanism to generate controls and investment in pollution abatement technologies (rather than optimality of some function, which expresses a particular point of view). These particular facets of our problem are more in tune with the practical setting and the environment within which polluters, regulators, and NGOs such Green Peace operate. To simplify our analysis, we consider first an intuitive solution of the environmental game under a “risk neutrality assumption". In this case, all random payoffs in the bimatrix game (Table 2) are treated by their expected values. Subsequently, issues associated to environmental risks are considered as well.

Denote the regulator's environmental control strategy by $0 \leq y \leq 1$. The polluting firm, however, reaches two decisions, regarding the pollution technology determining the probability of polluting and the preventive control efforts it exercises. Both are defined over probabilities continuum (x, p), $(0 \leq x \leq 1)$. Since the cost of employing a technology that does not pollute consistently, (equivalent to a "zero-defects" technology) implies $C(1) = \infty$, we presume that the cost of not polluting is not acceptable. However, pollution effects can be mitigated by preventive control efforts. Of course, both the propensity to pollute and the preventive control efforts are dependent, for the choice of one affects the other. For example, if p is very small, then it is possible that little preventive effort has to be exercised, and vice versa. Regulators may provide as well incentives for the firm to invest in pollution abatement equipment and technologies, as well as resort to greater efforts in preventive controls.

Using the defined game, under the assumption of risk neutrality, we obtain the bimatrix non zero sum game defined below:

	Control by Regulator	No controls by Regulator
Control by Firm	$\pi_P - [C(p) + c_F]$ $\pi_S - c_S$	$\pi_P - [C(p) + c_F]$ π_S
No Control by Firm	$\pi_P - C(p) - (1-\alpha)p\hat{\hat{\Phi}}$ $\pi_S - c_S - \alpha p\hat{\hat{\Phi}}$	$\pi_P - C(p)$ $\pi_S - p\tilde{\hat{\Phi}}$

Table 2: The (Polluter, Regulator) Risk Neutral Payoff Matrix

Where $\hat{\Phi}$ denotes the expected cost of a polluting event. The solution of this game is treated in [17] and therefore we shall only summarize the essential facets of its solution given by proposition 1 below.

Proposition 1: For risk a risk neutral polluting firm and a risk neutral regulator, the propensity to introduce preventive controls by the firm and the regulator to control the firm are given by the following:

(1) *If* $c_S \geq (1-\alpha)p\hat{\hat{\Phi}}$ *then* $y^* = 0$ *and in all cases* $y^* \neq 1$ *, 0<p<1.*

(2) $0 < x^* < 1$ *in all conditions*

(3) *If* $c_S \leq (1-\alpha)p\hat{\hat{\Phi}}$ *then for risk neutral firm and regulator:*

$$x^* = 1 - \frac{c_S}{(1-\alpha)p\hat{\hat{\Phi}}}$$
$$y^* = \frac{c_F}{(1-\alpha)p\hat{\hat{\Phi}}}$$

(4) *The Nash equilibrium values for both the polluting firm and the regulator are given by:*

(5) $V_F = \pi_P - C(p) - c_F; \quad V_S = \pi_S - \dfrac{c_S}{1-\alpha}.$

The implications of these results are noteworthy. Of course, we see that the effects of α, society's share in "cleaning environmental pollution" decreases the propensity for preventive controls by the firm and increases the propensity of the regulator to control the firm. By the same token, when the firm improves its pollution technology (p is smaller), then the firm will use less preventive control while the regulator will use more controls (to compensate the reduction in preventive efforts applied by the firm)! Other aspects are treated in detail in [17]. We shall, however, consider next the same game using a "satisficing solution" approach.

Environmental control and satisficing games

Based on our previous analysis, we have the following value functions:

$$\begin{aligned} V_F &= \pi_P - C(p) - c_F x - (1-\alpha)p\hat{\Phi}y(1-x) \\ V_S &= \pi_S - p\hat{\Phi} - (1-\alpha)p\hat{\Phi}xy + p\hat{\Phi}x - \left[c_S - (1-\alpha)p\hat{\Phi}\right] y \ . \end{aligned}$$

Let z_{ij} be the probability that (collaborating) choices (i,j) are made by the firm and the regulator respectively. In other words, z_{ij}, can be interpreted as the joint distribution of strategy choices with marginal distributions (x, y). In this case, assuming that strategy choices are dependent, we have:

$$\begin{array}{l} \sum_{j=1}^{2} z_{1j} = x, \ \sum_{j=1}^{2} z_{2j} = 1-x, \\ \sum_{i=1}^{2} z_{i1} = y, \ \sum_{i=1}^{2} z_{i2} = 1-y, \\ \sum_{i=1}^{2}\sum_{j=1}^{2} z_{ij} = 1, \qquad z_{ij} \geq 0 \ . \end{array}$$

This is in addition to the game constraints as we shall see below. Thus,

$$z_{ij} \begin{cases} = x_i y_j : & \text{if i and j are not coordinated} \\ \neq x_i y_j : & \text{if i and j are coordinated} \end{cases}$$

Let $\left(z_{ij}^*\right)$ be a satisficing solution with marginal distribution (x^*, y^*). Then, for our game, this marginal distribution in a no-coordination-game yields:

$$\max\left(x\log\frac{1}{x}\right)+\left((1-x)\log\frac{1}{1-x}\right)+y\log\frac{1}{y}+(1-y)\log\frac{1}{1-y}.$$

Subject to:

$$\begin{aligned}
&V_F \geq y\left(\pi_p-[C(p)+c_F]\right)+(1-y)\left(\pi_p-[C(p)+c_F]\right)\\
&V_F \geq y\left(\pi_p-C(p)-(1-\alpha)p\hat{\Phi}\right)+(1-y)\left(\pi_p-C(p)\right)\\
&V_S \geq x(\pi_S-c_S)+(1-x)(\pi_S-c_S-\alpha p\hat{\Phi})\\
&V_S \geq x(\pi_S)+(1-x)(\pi_S-p\hat{\Phi})\\
&0\leq x\leq 1,\ \ 0\leq y\leq 1
\end{aligned}$$

which can be written for convenience by:

$$\max\left(x\log\frac{1}{x}\right)+\left((1-x)\log\frac{1}{1-x}\right)+y\log\frac{1}{y}+(1-y)\log\frac{1}{1-y}.$$

Subject to:

$$\begin{aligned}
&(1-x)\left[c_F-(1-\alpha)p\hat{\Phi}y\right]\geq 0\\
&x(1-\alpha)p\hat{\Phi}y-c_F\geq 0\\
&(1-y)\left[c_S-(1-\alpha)p\hat{\Phi}(1-x)\right]\geq\\
&y\left[(1-\alpha)p\hat{\Phi}(1-x)-c_S\right]\geq 0\\
&0\leq x\leq 1,\ \ 0\leq y\leq 1\ .
\end{aligned}$$

This is a non linear optimization problem which can be solved by the usual Kuhn Tucker conditions (or numerically). In this case, there are nine possible solutions:

$$\begin{array}{l} x=x^*,y=y^*\\ x=x^*,y=0\\ x=x^*,y=1\\ x=0,y=y^*\\ x=1,y=y^*\end{array}\quad\text{and}\quad\begin{array}{l} x=0,y=0\\ x=1,y=1\\ x=1,y=0\\ x=0,y=1.\end{array}$$

Assume that there is an interior solution $x=x^*,y=y^*$, then, of course,

$$\begin{aligned}
&\left[c_F-(1-\alpha)p\hat{\Phi}y\right]\geq 0\ y\leq\tfrac{c_F}{(1-\alpha)p\hat{\Phi}}\text{ or }y=\tfrac{c_F}{(1-\alpha)p\hat{\Phi}}\\
&\left((1-\alpha)p\hat{\Phi}y-c_F\right)\geq 0\ y\geq\tfrac{c_F}{(1-\alpha)p\hat{\Phi}}\text{ or }y\geq\tfrac{c_F}{(1-\alpha)p\hat{\Phi}}
\end{aligned}$$

and therefore, there is one interior solution (the Nash solution):

$$y^* = \frac{c_F}{(1-\alpha)p\hat{\Phi}} \ , \quad x^* = 1 - \frac{c_S}{(1-\alpha)p\hat{\Phi}}.$$

The solution $x = x^*, y = 0$ is not possible, however, since the following contradicts the inequality constraints:

$$y \geq \frac{c_F}{(1-\alpha)p\hat{\Phi}}, \ x \leq 1 - \frac{c_S}{(1-\alpha)p\hat{\Phi}}.$$

The solution $x = x^*, y = 1$ is also impossible since it requires that $\left[c_F - (1-\alpha)p\hat{\Phi}\right] = 0$ while the solutions $x = 0, y = y^*$ point out as well to $c_S \geq (1-\alpha)p\hat{\Phi}$, as well as $c_S \leq (1-\alpha)p\hat{\Phi}$, which can be reached only when $c_S = (1-\alpha)p\hat{\Phi}$ and therefore this is also an unfeasible solution. When $x = 1, y = y^*$, an infeasible solution is obtained since $c_S \geq (1-\alpha)p\hat{\Phi}$, as well as $c_S \leq (1-\alpha)p\hat{\Phi}$, which implies that $c_S = (1-\alpha)p\hat{\Phi}$.

By the same token, except for $x = 1, y = 0 \rightarrow c_F \leq 0, c_S \geq 0$, which is not feasible, all the remaining solutions are feasible since:

$$\begin{aligned}
&x = 0, y = 0 \rightarrow c_F \geq 0, c_S \geq 0 \\
&x = 0, y = 1 \rightarrow c_F \geq (1-\alpha)p\hat{\Phi}, c_S \leq (1-\alpha)p\hat{\Phi} \\
&x = 1, y = 1 \rightarrow c_F \leq (1-\alpha)p\hat{\Phi}, c_S \leq (1-\alpha)p\hat{\Phi}.
\end{aligned}$$

In this case, we clearly note that there is a number of potential strategies and while the Nash solution is the most conservative one, it might be possible to satisfy the constraints implicit in a Nash equilibrium and select another solution with greater value. Below we summarize all the four feasible solutions and their values:

$$\begin{aligned}
&x = x^*, y = y^*, \ V_F = \pi_P - C(p) - c_F, \ V_S = \pi_S - \tfrac{c_S}{1-\alpha} \\
&x = 0, y = 0, \ V_F = \pi_P - C(p), \ V_S = \pi_S - p\hat{\Phi} \\
&x = 1, y = 1, \ V_F = \pi_P - C(p) - c_F, \ V_S = \pi_S - c_S \\
&x = 0, y = 1, \ V_F = \pi_P - C(p) - (1-\alpha)p\hat{\Phi}, \ V_S = \pi_S - c_S - p\hat{\Phi}\alpha.
\end{aligned}$$

As a result, a satisficing solution when $c_F \geq (1-\alpha)p\hat{\Phi}, c_S \leq (1-\alpha)p\hat{\Phi}$ is expressed in terms of a strategic choice regarding the following alternatives:

$$x = x^*, y = y^* \ ; \ x = 0, y = 0 \ ; \ x = 0, y = 1.$$

Since the interior solution (assuming no other information) is also unique, the entropy maximizing probability of selecting any of these three strategies is

$$P\left(x^{*},0\right)=\left(1/3,2/3\right);\ P\left(y^{*},0,1\right)=\left(1/3,1/3,1/3\right).$$

When $c_F \leq (1-\alpha)p\hat{\Phi}, c_S \leq (1-\alpha)p\hat{\Phi}$, the strategic choices are:

$$x=x^{*}, y=y^{*}\ ;\ x=0, y=0\ ;\ x=1, y=1$$

and the maximum entropy probabilities are:

$$P\left(x^{*},0,1\right)=\left(1/3,1/3,1/3\right);\ P\left(y^{*},0,1\right)=\left(1/3,1/3,1/3\right).$$

The joint probabilities are thus (say, for case $c_F \geq (1-\alpha)p\hat{\Phi}, c_S \leq (1-\alpha)p\hat{\Phi}$):

$$\begin{pmatrix} & y^{*} & 0 & 1 \\ x^{*} & 1/3 & 0 & 0 \\ 0 & 0 & 1/3 & 1/3 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

Generally, for a bi-matrix game with payoffs $(\mathbf{A},\mathbf{B})$, let the choice of such strategies be defined by probabilities z_{ij}. Maximum entropy probabilities in a "coordinated game" are thus given by solving:

$$\max\sum_{i=1}^{n}\sum_{j=1}^{m}\left(z_{ij}\log\frac{1}{z_{ij}}\right).$$

Subject to:

$$\begin{aligned}
&V_F=\sum_{i=1}^{n}\sum_{j=1}^{m}a_{ij}z_{ij}\geq\sum_{i=1}^{n}a_{ij}z_{ij},\ \ j=1,2,...m\\
&V_R=\sum_{i=1}^{n}\sum_{j=1}^{m}b_{ij}z_{ij}\geq\sum_{j=1}^{m}b_{ij}z_{ij},\ \ i=1,2,...n\\
&\sum_{j=1}^{m}z_{ij}=x_i,\ \ \sum_{i=1}^{n}z_{ij}=y_j,\ \ \sum_{i=1}^{n}\sum_{j=1}^{m}z_{ij}=1, z_{ij}\geq 0.
\end{aligned}$$

Note that $z_{ij}=x_iy_j$ if both players do not coordinate their game (i.e. they are independent). In this case, the problem to solve consists in solving:

$$\max\sum_{i=1}^{m}\left(x_i\log\frac{1}{x_i}\right)+\sum_{j=1}^{m}\left(y_j\log\frac{1}{y_j}\right)$$

Subject to:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j a_{ij} \geq \sum_{i=1}^{n} x_i a_{ij},\ j = 1, 2, ..., m$$
$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j b_{ij} \leq \sum_{j=1}^{m} y_i b_{ij},\ i = 1, 2, ..., n$$
$$\sum_{i=1}^{n} x_i = 1,\ \sum_{j=1}^{m} y_j = 1,\ x_i \geq 0,\ y_j \geq 0.$$

Of course, such problems can be compared to the Nash solution which is more conservative. The advantage of this approach, however, is that it may reveal other "acceptable" solutions in a satisficing sense. This is particularly the case for random payoffs game (as they recur in environmental games) for which Nash solutions may not be easily determined. For example, if we let the matrices entries be stochastic (as this is the case in realistic environmental games), and specify a satisficing solution based on a quantile (or Value at Risk, VaR, see also [30]) risk constraint, the search for a solution may be reduced to Quantile Risk Constraint games:

$$\max \sum_{i=1}^{m} \left(x_i \log \frac{1}{x_i} \right) + \sum_{j=1}^{m} \left(y_j \log \frac{1}{y_j} \right).$$

Subject to:

$$P\left(\sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j \tilde{a}_{ij} - \sum_{i=1}^{n} x_i \tilde{a}_{ij} \geq 0 \right) \geq 1 - \alpha,\ j = 1, 2, ..., m$$
$$P\left(\sum_{i=1}^{n}\sum_{j=1}^{m} x_i y_j \tilde{b}_{ij} - \sum_{j=1}^{m} y_i \tilde{b}_{ij} \leq 0 \right) \geq 1 - \beta,\ i = 1, 2, ..., n$$
$$\sum_{i=1}^{n} x_i = 1,\ \sum_{j=1}^{m} y_j = 1,\ x_i \geq 0, y_j \geq 0$$

where α and β denote the risk that each of the parties will be willing to assume in the (environmental) game.

Finally, a number of examples are solved below to highlight the difference between the Nash solution and the Entropy maximizing (satisficing) solution. Note that in our notation $\left(\begin{array}{cc} V_a, N_a & V_b, N_b \end{array} \right)$ denote the satisficing and Nash values of the game. Further, as can be seen, the satisficing values are always better or equal to the Nash solution.

$$(\mathbf{A}, \mathbf{B}) = \begin{pmatrix} 1,1 & 5,2 \\ 2,7 & 1,2 \end{pmatrix} \rightarrow \begin{pmatrix} .2802 & .3599 \\ .2879 & .0720 \end{pmatrix},$$
$$\left(\begin{array}{cc} V_a, N_a & V_b, N_b \end{array} \right) = \left(\begin{array}{cc} 2.725, 1.8 & 3.1593, 2 \end{array} \right)$$

$$(\mathbf{A},\mathbf{B}) = \begin{pmatrix} 1,1 & 1,0 \\ 1,0 & 1,2 \end{pmatrix} \rightarrow \begin{pmatrix} .3238 & .2571 \\ .1619 & .2572 \end{pmatrix},$$
$$\begin{pmatrix} V_a, N_a & V_b, N_b \end{pmatrix} = \begin{pmatrix} 1,1 & 0.8382, 0.666 \end{pmatrix}$$

$$(\mathbf{A},\mathbf{B}) = \begin{pmatrix} 1,1 & 0,2 \\ 0,7 & 1,0 \end{pmatrix} \rightarrow \begin{pmatrix} .4375 & .4375 \\ .0625 & .0625 \end{pmatrix},$$
$$\begin{pmatrix} V_a, N_a & V_b, N_b \end{pmatrix} = \begin{pmatrix} .5,.5 & 1.75, 1.75 \end{pmatrix}$$

$$(\mathbf{A},\mathbf{B}) = \begin{pmatrix} 100,41 & 72,82 \\ 50,75 & 100,54 \end{pmatrix} \rightarrow \begin{pmatrix} .1216 & .2171 \\ .2374 & .4329 \end{pmatrix}$$
$$\begin{pmatrix} V_a, N_a & V_b, N_b \end{pmatrix} = \begin{pmatrix} 82.05, 82.055 & 63.48, 63.48 \end{pmatrix}$$

$$(\mathbf{A},\mathbf{B}) = \begin{pmatrix} 1,0 & 0,0 \\ 0,0 & 1,0 \end{pmatrix} \rightarrow \begin{pmatrix} .25 & .25 \\ .25 & .25 \end{pmatrix},$$
$$\begin{pmatrix} V_a, N_a & V_b, N_b \end{pmatrix} = \begin{pmatrix} .5,.5 & .5,.5 \end{pmatrix}$$

$$(\mathbf{A},\mathbf{B}) = \begin{pmatrix} 1,1 & 1,0 \\ 0,1 & 1,1 \end{pmatrix} \rightarrow \begin{pmatrix} .4298 & .0 \\ 0 & .5202 \end{pmatrix},$$
$$\begin{pmatrix} V_a, N_a & V_b, N_b \end{pmatrix} = \begin{pmatrix} 1,1 & 1,1 \end{pmatrix}.$$

In conclusion, we can state the definition and the solution of games provide a wide range of interpretations and potential approaches for dealing with environmental games and calculating solutions that might be used for environmental negotiations. While the Nash solution concept has dominated the Game Theory literature, there may be cases where the calculation of the Nash solution might be over-conservative or too difficult to determine and implement. This is likely to be the case in complex and random payoffs game (as it is the case in the environmental games). In these circumstances, a concept of satisficing may be appropriate in determining a solution which might not meet the axiomatic foundations of traditional Nash-Games but may provide nonetheless some useful guidelines to implementing game solutions.

There are, of course, many facets to this problem, which could be considered and have not been considered. For example, risk aversion, the effects of risk sharing, the application of cooperative efforts (such as subvention of pollution abatement investments through tax incentives and their like in environment games [31, 32]). To simplify our analysis, we have used a quantile risk as a mechanism to quantify the risk exposure game participants may be willing to sustain. These are, therefore, many topics for further research. The basic presumption of the "satisficing" solution to environmental games, is that it is very difficult to fully enforce pollution prevention by firms, as a result, some risk controls are needed to ensure that firms be controlled so that appropriate efforts are carried by these firms. In such situations, the number of intervening variables is large and solutions based on Nash equilibria difficult to assess and to implement.

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