HAIT Journal of Science and Engineering, Series: Engineering **2** (1) , pp. xxx-xxx

Selection of data modulation techniques in spread spectrum systems using modified processing gain definition

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Received 18 July 2004, accepted 13 September 2004

Abstract

Limitations and ambiguities of conventional processing gain definition are revealed and resolved. Modified parameters, effective processing gain and relative effective processing gain, are introduced and analyzed. The relative effective processing gain is used for selecting data modulation techniques that provide maximum suppression of additive interference in spread spectrum communication systems.

PACS: 07.50.Qx, 84.40.Ua, 84.30.Qi

1 Introduction

Suppression of additive interference (SAI) in a spread spectrum (SS) communication system determines its jam resistance and multiple access capacity. A parameter called "processing gain" G_p was introduced as a measure of SAI more than 50 years ago [1]. Since spreading distributes a low-dimensional data signal in a high-dimensional space of the SS signal, the most general definition of G_p is [1–5]:

$$G_p = D_{ss}/D_d, \tag{1}$$

where D_{ss} and D_d are the dimensionalities of the SS signal and data at the modulator input, respectively. This definition requires clarification in some cases. Besides that, it reflects only a spreading factor of an SS system.

When G_p was introduced, there were few data modulation/demodulation and channel encoding/decoding techniques. Thus, the spreading factor practically alone determined the SS system SAI. At present, advanced data modulation/demodulation and encoding/decoding techniques significantly contribute to the system SAI [3–6]. To characterize adequately the contribution of data modulation/demodulation and encoding/decoding techniques, processing gain should reflect the energy required for sufficient performance of communication system in addition to the spreading factor G_p.

In this paper, the ambiguities of conventionally defined G_p are revealed and resolved (Sect. 2), modifications of the processing gain definition are proposed and analyzed (Sect. 3), and a data modulation technique that maximizes system SAI in a channel with additive white Gaussian noise (AWGN) is selected using a modified processing gain definition (Sect. 4). The analysis is performed for the processing structure presented in Fig. 1 within the framework of a direct sequence (DS) SS communication system. In general, the signals and operations reflected by the block diagram shown in Fig. 1 can be complex-valued.

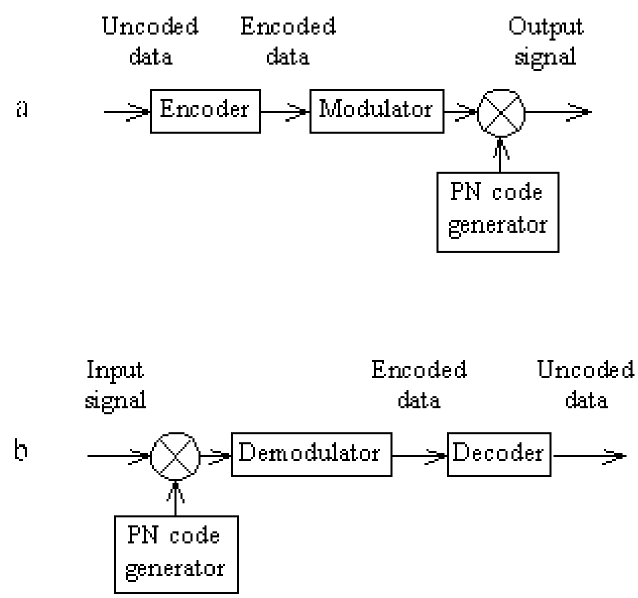

Figure 1: Simplified block diagram of the data processing structure: (a) transmitter part, (b) receiver part.

2 Resolution of G_P ambiguities

2.1 Influence of coordinate system

The ambiguities of conventionally defined processing gain G_p (i.e. spreading factor) are related to signal dimensionality D. This dimensionality is determined by the sampling theorem. According to this theorem, a signal with bandwidth B and duration T can generally be represented by $2BT$ its samples. Therefore, in general,

$$D = 2BT. \tag{2}$$

However, when additional limitations are imposed on the signal (for example, a certain type of modulation), the signal dimensionality can be reduced in some cases, i.e. D can be less than $2BT$. This should be taken into account when G_p is calculated.

The sampling theorem has many versions, and its different versions can be interpreted as different coordinate systems. It is easy to show that D depends on the coordinate system. For example, the unique discrete-time representation of M-ary phase shift keying (PSK) requires $2BT$ samples of its in-phase (I) and quadrature (Q) components, and only $BT + 1$ samples of its amplitude and phase. The question arises: what value of D should be used to determine G_p? To answer this question, recall that the signal energy is equal to the sum of the energies of its components exclusively in an orthogonal space. Consequently, only orthogonal discrete-time representations of signals can be used to determine D for G_p calculation. In the considered example, signal representation by the samples of its I and Q components is orthogonal while representation by the samples of its amplitude and phase is not. Therefore, $D = 2BT$ for M-ary PSK. It is important to note that there are many other orthogonal representations of signals besides their representation by the pairs of I and Q samples. For example, representation by the samples of the signal instantaneous values is also orthogonal. Representation by the samples of the signal I and Q components is widely used because this complex-valued baseband equivalent of a real-valued bandpass signal is convenient for many procedures of digital signal processing.

Thus, limiting signal discrete-time representation methods to the orthogonal ones allows us to resolve ambiguities related to the type of coordinate system. However, it does not resolve all ambiguities.

2.2 Difference between dimensionalities of data and spreading modulations

Let us show that dimensionalities can be different for the same modulation technique analyzed within the same coordinate system depending on whether it is used for data modulation or for spreading.

Usually binary PSK (BPSK), quaternary PSK (QPSK), or offset QPSK (OQPSK) is used for spreading. When BPSK is used, the same chips are sent over both I and Q channels if the same data are transmitted over these channels. In the case of QPSK or OQPSK, chips sent over I and Q channels can be different. Let us determine if the difference between BPSK, on the one hand, and QPSK or OQPSK, on the other, influences D_{ss}. Sending the same chips over I and Q channels means that appropriate rotation of I and Q axes allows concentrating the total signal energy only in one (I or Q) channel. In principle, this halves D_{ss}. However, the phase uncertainty introduced by the wireless channel makes it practically impossible to align the phases of intentional or unintentional interference and the signal at the receiver input. Therefore, the entire $2B_{ss}T$-dimensional space should be jammed, to disrupt communication. Here, B_{ss} is the spread spectrum signal bandwidth. Consequently, for all the spreading techniques mentioned above,

$$D_{ss} = 2B_{ss}T_s, \tag{3}$$

where T_s is the symbol length at the input of spreading stage (see Fig. 1). In (3), T is chosen equal to the symbol length T_s because the integration time in the data demodulator is equal to T_s. Thus, any choice of spreading modulation (BPSK, QPSK, or OQPSK) does not influence D_{ss} and the system SAI, although it influences the probabilities of detection and interception, side lobe regeneration, and other parameters of the SS system.

In contrast with spreading, the technique used for data modulation influences D_d because data modulation is synchronized with spreading, and there is no phase uncertainty within structure shown in Fig. 1a. Therefore, $D_d = 1$ for BPSK, and $D_d = 2$ for QPSK and OQPSK.

2.3 Spreading factors for data modulations with arbitrary alphabet sizes

Currently, nonbinary data modulation techniques with various alphabet sizes M are widely used in communications. When spreading factors of different modulation techniques with different M are compared, it is reasonable to perform the comparison for the same B_{ss} and bit rates $R_b = 1/T_b$.

Here, T_b is the bit length in the digital channel, which includes all the system units from the encoder output to the decoder input and the physical channel. To achieve the same R_b, it is sufficient to select $T_s = T_b \log_2 M$ for each modulation technique.

As shown in the previous subsection, data dimensionalities D_d are different for BPSK, on the one hand, and for QPSK and OQPSK, on the other. Generalizing this result, all data modulation techniques with arbitrary M can be divided into two groups. The first group includes all techniques that transmit the same data over both I and Q channels. For instance, BPSK, differential BPSK (DBPSK), and pulse amplitude modulation (PAM) belong to this group. For all the techniques of the first group, $D_d = 1$. The second group includes all modulation techniques that transmit different data over I and Q channels. QPSK, differential QPSK (DQPSK), OQPSK, and quadrature amplitude modulation (QAM) are examples of the techniques belonging to this group. For all the techniques of the second group $D_d = 2$. Taking into account (1), (3), clarifications provided above, and the fact that comparison of modulation techniques should be performed for the same B_{ss} and R_b, we get for the modulation techniques of the first group

$$\begin{aligned} G_{p1} &= D_{ss1}/D_{d1} = 2B_{ss}T_{s1}/1 = 2B_{ss}T_{s1} = 2B_{ss}T_b \log_2 M_1 \\ &= 2B_{ss}T_b k_1 = 2k_1(B_{ss}/R_b), \end{aligned} \tag{4}$$

and for the modulation techniques of the second group

$$\begin{aligned} G_{p2} &= D_{ss2}/D_{d2} = 2B_{ss}T_{s2}/2 = B_{ss}T_{s2} = B_{ss}T_b \log_2 M_2 \\ &= B_{ss}T_b k_2 = k_2(B_{ss}/R_b), \end{aligned} \tag{5}$$

where $k = \log_2 M$ is the number of bits per symbol.

2.4 Analysis of spreading factors for data modulations of the first and second groups

It follows from (4) and (5) that data modulation techniques of the second group should convey double the number of bits per symbol compared to the techniques of the first group, to achieve the same G_p for identical B_{ss} and R_b. In other words, $k_2 = 2k_1$is required in this case. For example, BPSK, DBPSK, QPSK, DQPSK, and OQPSK used for data modulation have the same G_p for identical B_{ss} and R_b because $k_1 = 1$ for BPSK and DBPSK, and $k_2 = 2$ for QPSK, DQPSK, and OQPSK. However, since $B_{ss}T_s$ cannot be smaller than 1, G_p cannot be less than 2 for BPSK and DBPSK, while QPSK, DQPSK, and OQPSK can have $G_p = 1$ when R_b is doubled. Likewise, PAM and QAM have the same G_p when the QAM alphabet size $M_2 = M_1^2$, where M_1 is the PAM alphabet size.

There are several ways to interpret (4) and (5). Two of them are considered below. The first interpretation is as follows. Let B_i denote the bandwidth of the integrator in the data demodulator (see Fig. 1b). Then, $T_s = T_b \log_2 M \approx 1/B_i$. This allows rewriting (4) and (5) for the modulation techniques of the first and second group respectively as follows:

$$G_{p1} = 2B_{ss}/B_{i1}, \tag{6}$$

$$G_{p2} = B_{ss}/B_{i2}. \tag{7}$$

The factor of 2 in (6) is caused by concentration of the total signal energy only in one channel. At the same time, its absence in (7) exposes the division of the signal energy between $I$and Q channels. Since B_{ss}/B_i is the ratio of the receiver input filter bandwidth to the integrator bandwidth, (6) and (7) reflect the fact that modulation techniques of the first group provide two times higher SAI than modulation techniques of the second group for the same B_i. This is not surprising because, for the same B_i, modulation techniques of the first group have two times lower bit rate than modulation techniques of the second group.

Another interpretation can be obtained by taking into account that $B_{ss} \approx 1/T_c$ where T_c is the chip length. Then, (4) and (5) respectively can be rewritten as follows:

$$G_{p1} = 2T_b \log_2 M_1/T_c = 2T_{s1}/T_c, \tag{8}$$

$$G_{p2} = T_b \log_2 M_2/T_c = T_{s2}/T_c. \tag{9}$$

The ratio T_s/T_c is a quantity of chips per symbol in each (Ior Q) channel. The factor 2 in (8) indicates that at each instant two chips (one in I channel and another in Q channel) belong to the same transmitted number (bit in the binary case) when modulation techniques of the first group are used. This factor is absent in (9) because simultaneous chips in I and Q channels belong to different transmitted numbers (bits in the binary case) when modulation techniques of the second group are used. When $T_{s1} = T_{s2}$, $G_{p1} = 2G_{p2}$. However, doubled G_p for modulation techniques of the first group is achieved at the expense of halved bit rate. Indeed, modulation techniques of the first group have two times lower bit rate than the techniques of the second group for the same T_s.

The examples above illustrate the fact that modulation techniques of the first and second groups have the same G_p when $k_2 = 2k_1$ and B_{ss} and R_b are identical for both groups. Does it mean that modulation techniques of both groups have the same SAI in this situation? The answer is negative. In most cases (but not always), modulation techniques of the second group allow increase in their Euclidean distances by better disposition of signal points within the signal constellations. This can be illustrated, for example, by comparison of PAM and QAM. However, conventionally defined processing gain G_p does not reveal this fact.

3 Modification of processing gain definition

3.1 Effective processing gain

Even comparative analysis of spreading factors for data modulation techniques of the first and second groups performed above shows that G_p does not reflect all the factors that determine the system SAI. To take into account all these factors, processing gain definition should be modified, to reflect the combined influence of encoding/decoding, modulation/demodulation, and spreading/despreading (see Fig. 1). It is reasonable to designate a parameter characterizing the system SAI as an effective processing gain G_e. Since the factors that influence the system SAI are multiplicative,

$$G_e = G_c \cdot G_m \cdot G_p, \tag{10}$$

where G_c is a coding gain, G_m is a modulation gain, and G_p is the conventionally defined processing gain (i.e. a spreading factor) determined by (4) and (5). G_c characterizes the increase in the system SAI provided by the encoding/decoding technique. G_m characterizes the increase in the system SAI provided by the improvement in the data modulation/demodulation technique. Sometimes, it is more convenient to express the effective processing gain in dB. In this case, (10) can be rewritten as follows:

$$G_{e(dB)} = G_{c(dB)} + G_{m(dB)} + G_{p(dB)}, \tag{11}$$

where $G_{e(dB)} = 10{\cdot}\log G_e$, $G_{c(dB)} = 10{\cdot}\log G_c$, $G_{m(dB)} = 10{\cdot}\log G_m$, and $G_{p(dB)} = 10{\cdot}\log G_p$.

Although (10) and (11) reflect separately estimated factors that determine the system SAI, the separation is not always possible. Indeed, in some cases, modulation cannot be separated from spreading. For example, when data modulation is transmission of one out of L PN sequences, modulation and spreading are inseparable. In other cases, this separation is possible, but the spreading factor can impose some limitations on the selection of data modulation techniques and their parameters. Encoding/decoding and modulation/demodulation can also be tightly connected. It happens, for example, when demodulators make soft decisions or trellis-coded modulation is used. In some cases, connections among spreading, modulation, and encoding exist but they are indirect. Indeed, spreading, data modulation, and the physical channel determine the digital channel type. Selection of the optimal channel codes depends on the types of digital channels. Thus, modulation and spreading influence the selection of channel codes. When separate estimation of G_c, G_m, and G_p or any pair of these three factors is impossible, they should be estimated jointly.

When separation of encoding and modulation is possible, the estimates of $G_{c(dB)}$ for various encoding/decoding techniques can be found in publications [5, 6] and others. For example, the approximate estimate of the upper bound of $G_{c(dB)}$ for linear block codes is provided in [5]:

$$G_{c(dB)} = 10 \cdot \log(R_c d_{min} - l \cdot \ln 2/\gamma_b), \tag{12}$$

where R_c is the code rate, d_{min} is the minimum Hamming distance of the code, and γ_b is a signal-to-noise ratio (SNR) per bit for the uncoded data. $R_c = l/n$, where l is the number of information bits per block, and n is the total number of bits per block. $\gamma_b = E_{b1}/N_0$, where $E_{b1}$is the energy per bit of the uncoded data, and N_0 is one-sided noise power spectral density. The estimate (12) shows that $G_{c(dB)}$ depends on the type of code, which determines d_{min} for given l and n, as well as on γ_b. The decoding technique is considered optimal in this case. The quantitative estimates of G_c for many codes can be found, for example, in [6].

Channel encoders and decoders deal with two different bit rates: the channel bit rate R_b (at the encoder output and decoder input) and the uncoded data bit rate R_{b1} (at the encoder input and decoder output). They are connected by equation $R_{b1}/R_b = R_c$. At the same time, modulators and demodulators deal only with the channel bit rate R_b. Even if the symbol rates at the input and the output of both modulator and demodulator are different, the bit rate is the same.

Before introducing the modulation gain $G_{m(dB)}$, note that $G_{c(dB)}$ and $G_{p(dB)}$ can be considered relative parameters. Indeed, $G_{c(dB)}$ reflects the improvement in the system SAI relative to the uncoded data, and $G_{p(dB)}$ reflects the improvement relative to the unspread signal. As to the data modulation/demodulation technique, its influence on the system SAI, reflected by $G_{m(dB)}$, can be determined only relative to a reference technique.

Coherently demodulated BPSK is the best reference because it is the most energy-efficient binary modulation/demodulation technique. Taking into account that the system SAI is inversely proportional to the energy per bit E_b needed to maintain an acceptable bit error probability P_b for a given interference, G_m and $G_{m(dB)}$ can be introduced as follows:

$$G_m = E_{b.r}/E_{b.a} \text{ and } G_{m(dB)} = 10 \cdot \log E_{b.r} - 10 \cdot \log E_{b.a}, \qquad (13)$$

where $E_{b.a}$ and $E_{b.r}$ are the energies per bit required to maintain the same P_b using the analyzed and the reference (coherently demodulated BPSK) modulation/demodulation techniques, respectively.

3.2 Relative effective processing gain

When comparison of only data modulation techniques is required, it is reasonable to assume $G_c = 1$ and estimate jointly the gain provided by data modulation and spreading. The latter allows comparing all modulation techniques, including techniques that are inseparable from spreading. Thus, (10) can be simplified:

$$G_e = G_m \cdot G_{p.a}, \qquad (14)$$

where $G_{p.a}$ is a spreading factor G_p for the analyzed data modulation technique. After multiplying and simultaneously dividing the right-hand side of (14) by $G_{p.r}$, (14) can be rewritten as follows:

$$G_e = (G_m \cdot G_{p.a}/G_{p.r}) \cdot G_{p.r} = G_r \cdot G_{p.r}, \qquad (15)$$

where $G_{p.r}$ is a spreading factor G_p for the reference data modulation technique (BPSK), and $G_r = G_m \cdot G_{p.a}/G_{p.r}$ is a relative effective processing gain provided by the analyzed data modulation technique combined with spreading compared to the gain of the reference modulation (BPSK) also combined with spreading for the same B_{ss} and R_b. As follows from (4) and (5), $G_{p.r} = 2B_{ss}T_b$ and $G_{p.a} = 2B_{ss}T_b\log_2 M_a/D_{d.a}$, where M_a is the alphabet size of the analyzed modulation. Taking into account this and (13), we get

$$G_r = E_{b.r}\log_2 M_a/(E_{b.a}D_{d.a}), \qquad (16)$$

$$G_e = 2B_{ss}T_b E_{b.r}\log_2 M_a/(E_{b.a}D_{d.a}). \qquad (17)$$

Relative effective processing gain G_r defined by (16) is convenient for comparison and selection of modulation/demodulation techniques, and effective processing gain G_e defined by (17) is an adequate measure of the SS system SAI. Both G_r and G_e can be expressed in dB: $G_{r(dB)} = 10{\cdot}\log G_r$

and $G_{e(dB)} = 10 \cdot \log G_e$. $G_{r(dB)}$ can be positive or negative. A negative $G_{r(dB)}$ means that an analyzed data modulation technique combined with spreading provides smaller SAI than the reference technique.

Equations (16) and (17) can be used for comparison of modulation/demodulation techniques and estimation of the system SAI respectively in communication systems operating in all types of channels. Taking into account (10)-(12), they can be generalized to incorporate the influence of encoding/decoding. They can also be simplified and used for analysis of communication systems without spreading.

4 Selection of modulated techniques

4.1 Preliminary consideration

Application of G_r can be illustrated by its use for selecting a modulation technique that after being combined with coherent demodulation provides the highest SAI in a channel with additive white Gaussian noise (AWGN). To achieve the highest SAI without reducing system throughput, this selection is performed for the same $B_{ss}$and R_b for all analyzed techniques. Since encoding is not considered, equation (16) is used. According to (16), the system SAI can be improved only by increasing M_a and/or reducing $E_{b.a}$ for given $B_{ss}$and R_b. Reduction of $D_{d.a}$ does not increase G_r because it reduces R_b. Since the reference technique (coherently demodulated BPSK) has the highest noise immunity among binary modulation/demodulation techniques, the desired modulation has to be sought among the techniques that allow extending M_a. The search is performed in two steps. At the first step, the most promising bandwidth-efficient technique and the most promising energy-efficient technique are selected. At the second step, they are compared to each other.

Since bandwidth-efficient techniques (M-ary PSK, PAM, and QAM) have approximately the same bandwidth-efficiency, QAM, which is the most energy-efficient among them, has the highest G_r. The differences in the energy efficiencies among the energy-efficient techniques (simplex keying, orthogonal keying, and biorthogonal keying) are negligible for substantial M [4, 5, 7, 8]. Simultaneously, the bandwidth efficiency of biorthogonal signals is noticeably higher than that of the others. Therefore, biorthogonal keying is the most promising among energy-efficient techniques. Thus, the selection of a modulation technique that provides the highest SAI can be reduced to the comparison of QAM and biorthogonal keying. Since both QAM and biorthogonal keying transmit different data over I and Q channels, they belong to the modulation techniques of the second group. For that reason, (16) can be rewritten:

$$G_r \;= [E_{b.r}/(2E_{b.a})] \cdot \log_2 M. \tag{18}$$

4.2 Analysis of QAM

Equations for P_b in AWGN channels can be used to determine $E_{b.r}$ and $E_{b.a}$. For coherently demodulated BPSK and QPSK [4, 5]

$$P_b = Q(\sqrt{2E_b/N_0}), \tag{19}$$

where $Q(x)$ is the co-error function. Note that BPSK is the reference data modulation, and QPSK can be considered the simplest case of QAM with $M_a = 4$. For coherently demodulated QAM with a square constellation [4, 9],

$$P_b \approx \frac{4(1 - M^{-0.5})}{\log_2 M} \cdot Q\left[\sqrt{\frac{1.5 \log_2 M}{M-1} \cdot (2E_{bav}/N_0)}\right], \tag{20}$$

where E_{bav} is the average energy of one bit.

Table 1, calculated according to (18)-(20) for $P_b = 10^{-6}$, shows that QAM and consequently all other bandwidth-efficient modulation techniques do not allow increasing $G_{r(dB)}$ by extending the system alphabet. The reason is that reduction of their Euclidean distances is faster than increase in their throughput as M grows. The use of trellis coding [4, 5, 10] together with QAM increases $G_{r(dB)}$ due to the coding gain, which is automatically included in $G_{r(dB)}$ in this case. However, because of the necessity to limit the complexity of decoding algorithm, the coding gain provided by trellis coding cannot exceed 3...6 dB in practical situations [4, 10]. Thus, even combining the most promising bandwidth-efficient modulation (QAM) with trellis coding can only decelerate reduction of the system SAI caused by extending the alphabet.

M	4	16	64	256	1024	4096
k	2	4	6	8	10	12
$G_{r(dB)}$	0	-4	-8.3	-13	-18	-23.2

Table 1: Values of M, k, and $G_{r(dB)}$ for QAM.

4.3 Analysis of biorthogonal keying

As shown above, biorthogonal keying is the most promising technique among energy-efficient data modulations. Both versions of biorthogonal keying (biorthogonal phase-frequency shift keying and biorthogonal waveform coding) have the same limitation and the same P_b in an AWGN channel. Thus, it is sufficient to analyze only biorthogonal waveform coding, in which biorthogonal codes modulate a carrier using QPSK. The alphabet sizes of all energy-efficient techniques including biorthogonal keying are limited by the ratio B_{ss}/R_b. Therefore, it is necessary to determine relations between this ratio and the number of bits per symbol k for biorthogonal signals first.

Since biorthogonal keying belongs to the modulation techniques of the second group, the ratio B_{ss}/R_b is equal to the number of chips in each bit carried by a biorthogonal symbol. Each bit should contain at least one chip. Therefore, the maximum throughput is achieved when the numbers of bits and chips per symbol coincide, i.e. $B_{ss}/R_b = 1$. When the system alphabet size is equal to $M = 2^k$, the minimum number of chips per symbol is equal to k. These k chips allow generation of $2k$ different biorthogonal signals. Since M cannot exceed the number of different biorthogonal signals that can be generated,

$$2^k \le 2k \text{ when } B_{ss}/R_b = 1. \tag{21}$$

As follows from (21), the maximum size of the system alphabet is $M = 4$ ($k = 2$) when $B_{ss}/R_b = 1$. This means that QPSK is the simplest type of biorthogonal keying. In general, the number of chips per symbol is equal to $k{\cdot}\text{int}(B_{ss}/R_b)$. This number of chips allows generation of $2k{\cdot}\text{int}(B_{ss}/R_b)$ different biorthogonal signals. Therefore, the maximum value of k (and consequently the maximum value of $M = 2^k$) for any given B_{ss}/R_b can be obtained from the following inequality:

$$2^k \le 2k \cdot \text{int}\,(B_{ss}/R_b). \tag{22}$$

It follows from (22) that the minimally required ratio B_{ss}/R_b for given k (and consequently M) is:

$$B_{ss}/R_b = 2^{k-1}/k. \tag{23}$$

The upper bound of P_b for equally likely and equal-energy biorthogonal signals with $M > 8$ in an AWGN channel is as follows [4, 7]:

$$P_b \le \frac{1}{2}\left[(M-2)Q\left(\sqrt{\frac{E_b(\log_2 M)}{N_0}}\right) + Q\left(\sqrt{\frac{2E_b(\log_2 M)}{N_0}}\right)\right]. \tag{24}$$

This upper bound becomes increasingly tight as the signal-to-noise ratio E_b/N_0 grows.

Table 2 presents the values of B_{ss}/R_b and $G_{r(dB)}$ for biorthogonal keying with various M and k. The ratio B_{ss}/R_b was calculated according to (23). G_r was calculated according to (18) because biorthogonal keying belongs to the modulation techniques of the second group. Equations (19) and (24) were used to calculate the values of $E_{b.r}$ and $E_{b.a}$ for $P_b = 10^{-6}$ in (18). Since fairly high signal-to-noise ratios correspond to $P_b = 10^{-6}$, equation (24) gives a practically accurate values of $E_{b.a}$.

M	4	16	64	256	1024	4096
k	2	4	6	8	10	12
B_{ss}/R_b	1	2	5.3	16	51.2	170.7
$G_{r(dB)}$	0	5.2	8.3	10.6	12	13.3

Table 2: Values of M, k, and $G_{r(dB)}$ for biorthogonal signals.

4.4 Comparison of the modulation techniques

The results presented in Tables 1 and 2 show that, in contrast with QAM, biorthogonal keying allows increasing $G_{r(dB)}$ by extending the system alphabet M. It is also clear that biorthogonal keying provides the highest system SAI in an AWGN channel. At the same time, although $G_{r(dB)}$ monotonically increases as M grows (see Table 2), extending M beyond $M = 256$ ($k = 8$) is not always practical even if a given ratio B_{ss}/R_b makes it possible. The reason is the diminishing return in the system SAI and fast growing complexity of demodulation as k is increased further than $k = 8$.

It is known that orthogonal keying with noncoherent demodulation has practically the same noise immunity as biorthogonal keying with coherent demodulation when $k > 7$ [4, 8]. Taking into account that noncoherent demodulation significantly simplifies synchronization and increases its speed, the use of orthogonal keying with noncoherent demodulation instead of biorthogonal one can be preferable in some cases if the ratio B_{ss}/R_b is sufficient. To determine what value of B_{ss}/R_b is sufficient, recall that the same chips should be transmitted over I and Q channels when noncoherent demodulation of orthogonal keying is used. Therefore, $k \cdot \text{int}(B_{ss}/R_b)$ chips per symbol allow generation of only $0.5k \cdot \text{int}(B_{ss}/R_b)$ different orthogonal signals. Consequently, the minimum ratio B_{ss}/R_b sufficient for generation of $M = 2^k$ different orthogonal signals can be obtained from the equation:

$$B_{ss}/R_b \ = \ 2^{k\,+\,1}/k. \tag{25}$$

Comparison of (23) and (25) shows that orthogonal keying with noncoherent demodulation requires four times wider bandwidth than biorthogonal keying with the same R_b. (Note that orthogonal keying with coherent demodulation requires only two times wider bandwidth than biorthogonal keying.) However, when high speed and/or reduced complexity of synchronization are required, transition from biorthogonal keying to orthogonal keying with noncoherent demodulation is advantageous if $B_{ss}/R_b \geq 2^{k\,+\,1}/k$ and the system SAI is already sufficient.

The results of the signal selection in an AWGN channel can be used in multipath channels only if G_p sufficiently suppresses the effects of multipath propagation. Otherwise, multipath propagation reduces the Euclidean distances of biorthogonal signals. Therefore, other modulation techniques with smaller Euclidean distances in a single-path channel but with better autocorrelation properties can provide higher G_r in multipath channel. Complementary code keying is an example of such a technique [11].

5 Conclusions

Ambiguities of the conventionally defined processing gain G_p, which is nothing more than a spreading factor, can be resolved by taking into account the following. Only orthogonal discrete-time representations can be used to determine the signal dimensionality for G_p calculation. For any spreading technique (BPSK, QPSK, or OQPSK) $D_{ss} = 2B_{ss}T_s$. Thus, spreading techniques do not influence G_p, although they influence the probabilities of detection and interception, side lobe regeneration, and other parameters of the SS system. Data modulation techniques influence the data dimensionality D_d. All data modulation techniques with arbitrary alphabet sizes can be divided into two groups. The techniques of the first group transmit the same data over both I and Q channels, and $D_d = 1$ for all of them. The techniques of the second group transmit different data over both I and Q channels, and $D_d = 2$ for all of them. Spreading factor G_p is defined by equations (4), (6), and (8) for the techniques of the first group and by equations (5), (7), and (9) for the techniques of the second group. Although modulation techniques of the first and second groups have the same G_p when $k_2 = 2k_1$ and their B_{ss} and R_b are identical, they may have different SAI. In most cases (but not always), modulation techniques of the second group have higher SAI because they enable an increase in Euclidean distances by better disposition of signal points within the signal constellations.

The effective processing gain G_e, introduced as an adequate measure of the system SAI, is a product of the coding gain G_c, modulation gain G_m, and spreading factor G_p. When separate estimation of G_c, G_m, and G_p or any pair of these three factors is impossible, they should be estimated jointly. G_m can characterize the improvement in the system SAI provided by a modulation/demodulation only relative to a reference technique. Coherently demodulated BPSK is the best reference because it is the most energy-efficient binary modulation/demodulation technique.

A new parameter, relative effective processing gain G_r, is introduced to simplify comparison and selection of data modulation techniques. This parameter reflects the relative gain provided by analyzed data modulation combined with spreading compared to that of the reference modulation (BPSK) also combined with spreading for the same B_{ss} and R_b. G_r can be calculated according to (16), while G_e, which is used to estimate the absolute value of the SS system SAI, can be calculated according to (17). Equations (16) and (17) can be applied to communication systems operating in all types of channels. They can easily be generalized to incorporate the influence of encoding/decoding. They can also be simplified for communication systems without spreading.

The use of G_r as a criterion for selecting data modulation techniques in an AWGN channel has shown the following. For given $B_{ss}$and R_b, only extending the system alphabet M and/or reducing the required energy per bit can improve the SS system SAI. Bandwidth-efficient modulation techniques cannot improve the system SAI because reduction of their Euclidean distances is faster than increase in their throughput as M grows. Energy-efficient modulation techniques improve the system SAI by extending M, and biorthogonal keying provides the best SAI in AWGN channels. Extending M monotonically increases G_r for biorthogonal signals. However, the growth of G_r is slower than the growth of the demodulation complexity. Therefore, extending M beyond $M = 256$ ($k = 8$) is not always practical even if a given ratio B_{ss}/R_b allows it. When $B_{ss}/R_b \geq 2^{k+1}/k$ and further extending M is impractical, transition from biorthogonal keying to orthogonal keying with noncoherent demodulation can be advantageous because it increases speed and reduces complexity of synchronization practically without reduction of the system SAI.

The results obtained for AWGN channels can be extended to multipath channels only if G_p sufficiently suppresses the effects of multipath propagation. Otherwise, multipath propagation reduces the Euclidean distances of biorthogonal signals. Therefore, other modulation techniques with smaller Euclidean distances in a single-path channel but with better autocorrelation properties can surpass biorthogonal keying.

References

[1] M.K. Simon, J.K. Omura, R.A. Scoltz, and B.K. Levit, *Spread Spectrum Communications Handbook* (McGraw-Hill, N.Y., Revised ed., 1994).

[2] R.L. Peterson, R.E. Ziemer, and D.E. Borth, *Introduction to Spread Spectrum Communications* (Prentice-Hall, Upper Saddle River, NJ, 1995).

[3] A.J. Viterbi, IEEE Communications Magazine, May 34 (2002).

[4] B. Sklar, *Digital Communications* (Prentice-Hall, Upper Saddle River, NJ, 2^{nd} ed., 2001).

[5] J.G. Proakis, *Digital Communications* (McGrow-Hill Higher Education, Boston, 4^{th} ed., 2001).

[6] G.C. Clark, Jr. and J.B. Cain, *Error-Correction Coding for Digital Communications* (Plenum Press, N.Y., 1981).

[7] W.C. Lindsey and M.K. Simon, *Telecommunication Systems Engineering* (Prentice-Hall, Englewood Cliffs, NJ, 1973).

[8] A.J. Viterbi, *Principles of Coherent Communications* (McGrow-Hill Book Company, N.Y., 1966).

[9] I. Korn, *Digital Communications* (Van Nostrand Reinhold Co., N.Y., 1985).

[10] G. Ungerboeck, IEEE Communications Magazine **25**, February 5 (1987).

[11] K. Halford, S. Halford, M. Webster, and C. Andren, *Proceeding of the ISCAS 1999*, Orlando, FL, USA, p. IV-427 (1999).