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Classical versus quantum percolation in quantum Hall systems in presence of nuclear polarization

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Abstract

We study localization-delocalization transition in quantum Hall systems with a random field of nuclear spins acting on two-dimensional (2d) electron spins via hyperfine contact (Fermi) interaction. We use Chalker-Coddington network model, which corresponds to the projection onto the lowest Landau level. The inhomogeneous nuclear polarization acts on the electrons as an additional confining potential, and, therefore, introduces additional parameter p (the probability to find a polarized nucleus in the vicinity of a saddle point of random potential) responsible for the change from quantum to classical behavior. In this manner we obtain two critical exponents corresponding to quantum and classical percolation. We also study how the 2d extended state develops into the one-dimensional (1d) critical state.

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1 Introduction

Celebrated quantum Hall effect (QHE) is realized in a 2d electron gas subjected to a strong perpendicular magnetic field and a random potential [1, 2]. The uniqueness of this phenomenon is in high precision of the plateaux in the Hall component and very rich physics in the interplateau transitions [3]. Here we will study the influence of the nuclear spin fields [4] on the critical exponents in QHE.

The physics of the random potential in quantum Hall systems could be roughly divided into spin independent and spin dependent (spintronics) electron scattering processes. Magnetic impurities perturb the QHE transport very strongly and will not be considered here. Recently sharply growing attention was attracted to the physics of the hyperfine interactions in the QHE. It was suggested theoretically [4] and observed experimentally [5, 6] that the underlying nuclear spin structure can provide the microscopic information on the 2d electron wave functions and provide strong influence on the precision and other parameters of a QHE system.

2 Random Hyperfine fields and nuclear spin relaxation rates

The interaction between electron and nuclear spins in heterojunctions under QHE conditions is due, usually, to the *hyperfine Fermi contact* interaction [7, 4]. This interaction is represented by the Hamiltonian:

$$\hat{H}_{int} = -\gamma_n \hbar \vec{I}_i \cdot \vec{H}_e, \tag{1}$$

where γ_n is the nuclear gyromagnetic ratio, $\vec{I}_i$ is the nuclear spin and $\vec{H}_e$ is the magnetic field on the nuclear site, produced by electron orbital and spin magnetic moments:

$$\vec{H}_e = -g\beta \sum_e \frac{8\pi}{3} \hat{s}_e \delta\left(\vec{r}_e - \vec{R}_i\right). \tag{2}$$

Here $\vec{r}_e$ is the electron radius-vector, $\hat{s}_e$ is the electron spin operator, $\beta = e\hbar/m_0 c$ is the Bohr magneton, g is the electronic g-factor and $\vec{R}_i$ is the nucleus radius-vector.

It follows from Eqs. (1) and (2), that once the nuclear spins are polarized, i.e. if $\left\langle \sum_i \vec{I}_i \right\rangle \neq 0$, the charge carriers spins feel the effective, time-dependent hyperfine field $B_{hf} = B^o_{hf} \exp\left(-t/T_1\right)$ (T_1 is a nucleus relxation time) which lifts the spin degeneracy even in the absence of external

magnetic field. In GaAs/AlGaAs one may achieve the spin splitting due to hyperfine field of the order of the Fermi energy [5, 6].The inhomogeneous nuclear polarization acts on the electrons as additional (to the scalar potential of the impurities) confining potential $V_{hf} = -\mu_B B_{hf}$ [8].

The nuclear spin polarization, once created, remains finite for macroscopically long times. Intensive experimental studies [5, 6] of this phenomenon in QHE systems have provided a more detailed knowledge on the hyperfine interaction between the nuclear and electron spins in heterojunctions and quantum wells. It was observed that the nuclear spin relaxation time is rather long (up to 10^3 sec) and the hyperfine field acting on the charge carriers spins is extremely high, up to $10^4 G$ [5].

Iordanskii et al. [9] have studied nuclear spin relaxation taking into account the creation of spin-excitons [10] in the flip-flop process. The energy for the creation of a spin-exciton can be provided by the long range impurity potential in a process, where the electron turns its spin while its center of orbit is displaced to a region with lower potential energy.

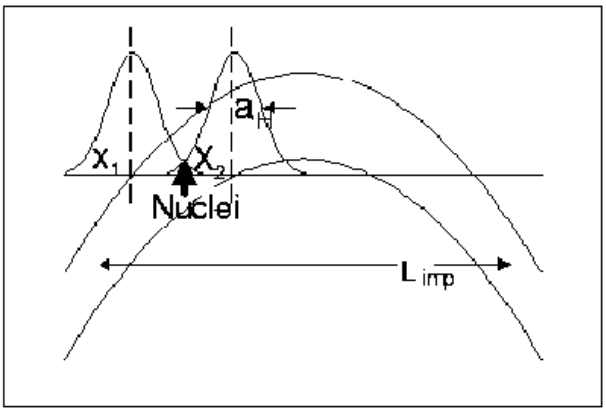

Figure 1: Long-range electrostatic potential, created by a remote impurity, provides the energy to reverse the electron spin in the nuclear-electron flip-flop process.

As shown in Fig. 1, the overlap of the initial and final location of the electron wave functions, centered at x_1 and x_2 respectively, is: $exp[-(x_1 - x_0)^2/a_H - (x_2 - x_0)^2/a_H^2]$. Here x_0 is the nuclear position. Nuclear spin relaxation by the conduction electron spin in the vicinity of a potential fluctuation is effective when the nuclear spin is positioned in the region of the overlapping initial and final states of the electron wave function.

The energy conservation in the spin-exciton creation process can be written in the form:

$$\mu_B g H_0 + E(p) = x \nabla U.$$

This expression defines the gradient of electric potential caused by the impurity, sufficient to create a spin-exciton during a flip-flop process. The probability of finding such a fluctuation is exponentially small: $exp[-(\nabla U)^2/2 < \nabla U^2 >]$.

The momentum $\mathbf{p}$ of the spin-exciton is small and therefore the expansion in p can be performed everywhere in the final expression for electronic density of states (DOS):

$$\text{DOS} = \int ImG(E, x, x) dx = e^{-S}. \tag{3}$$

Here S depends on the following combination of physical parameters:

$$S \propto |g| \frac{(\mu H E_c)}{R"(0)} \frac{L^2}{a_H^2}, \tag{4}$$

$R"(r) \equiv d^2R(r)/dr^2$ reaches the maximum, usually, at $r = 0$.

A complete expression for the nuclear depolarization rate from the golden rule formula at T=0 [9] is given by

$$T_1^1 \propto \nu_\uparrow (1 - \nu_\downarrow) \frac{1}{2\pi^2} \int ImG(k, \omega) \delta(\omega - \omega_N) L_n\left(k^2\right) e^{\frac{-k^2}{2}} d^2k d\omega,$$

where $\nu_\uparrow, \nu_\downarrow$ are filling factors for the electrons with spin up and spin down, respectively, n is the highest occupied Landau level and $L_n(k^2/2)$ are the Laguerre polynomials. Here the system of units is used, where $a_H^2 = c\hbar/eH = 1$ and $\hbar = 1$.

The nuclear relaxation time depends strongly on the vicinity to the impurity and its sign [3]. The presence of the impurity (long range potential) provides the necessary energy conservation in the spin-exciton creation process leading to the nuclear spin relaxation. We can, therefore, expect the following scenario: nuclear spins being polarized by some external field will then relax differently depending on whether they are close to maxima or minima of the scalar potential created by impurities. Therefore, they should affect strongly tunneling of electrons through saddle-point potential.

When random potential varies smoothly (its correlation length is much larger than the magnetic length as, e.g., in GaAs heterostructures) a semiclasscial description becomes relevant: electrons move along the lines of

constant potential. When two equipotential lines come close to each other (near a saddle point) tunneling is feasible. In this paper we investigate how this picture will be affected by strong nuclear polarization. We find that scaling of the localiztion length is modified (Eq. (12)), which is the main result of this work.

3 Network model

In the network model [11], electrons move along unidirectional links forming closed loops in analogy with semiclassical motion on contours of constant potential. Scattering between links is allowed at nodes in order to map tunneling through saddle point potentials. Propagation along links yields a random phase ϕ, thus links are presented by diagonal matrices with elements in the form $\exp(i\phi)$. Transfer matrix for one node relates a pair of incoming and outgoing amplitudes on the left to a corresponding pair on the right; it has the form

$$\mathbf{T} = \begin{pmatrix} \sqrt{1+\exp(-\pi\epsilon)} & \exp(-\pi\epsilon/2) \\ \exp(-\pi\epsilon/2) & \sqrt{1+\exp(-\pi\epsilon)} \end{pmatrix}. \tag{5}$$

In order for a system to be invariant, on average, under 90° rotation the transmission and reflection at the next neighbor node are interchanged, i.e. the transfer matrix has the same as in Eq. (5) form with a parameter $\epsilon' = -\epsilon$ [11]. In order to obtain this relation one simply interchanges Z_3 and Z_4 (see Fig. 2) and brings a new transfer matrix to the form of Eq.(5) . We therefore describe scattering at the nodes indicated in Fig. 1 by circles with transfer matrix $\mathbf{T}(\epsilon)$ and at the nodes indicated by boxes with $\mathbf{T}(-\epsilon)$.

The node parameter ϵ is a relative distance between the electron energy and the barrier height. It is related to the physical quantities descibing the system

$$\epsilon \equiv (E - (n+\frac{1}{2})E_2 - V_0)/E_1, \tag{6}$$

where E_1 measures the ratio between saddle-point paramters and magnetic field, E_2 is a distance between Landau levels at strong magnetic fields, and V_0 is a reference point of a scalar potential [[12]]

$$E_1 = \left[\Omega\left\{\gamma^2 + \left(\frac{\omega_c}{4}\right)^2\right\}^{1/2} - \frac{1}{4}\Omega^2 - \left(\frac{\omega_c}{4}\right)^2\right]^{1/2}, \tag{7}$$

where $\Omega = (\frac{1}{4}\omega_c^2 + (U_y - U_x)/m)^{1/2}, \gamma = (U_y + U_x)/2m\Omega$, and $U_{x,y}$ are the ciurvatures of the saddle-point potential. The oscillator frequency E_2 is

$$E_2 = \left[\Omega\left\{\gamma^2 + \left(\frac{\omega_c}{4}\right)^2\right\}^{1/2} + \frac{1}{4}\Omega^2 + \left(\frac{\omega_c}{4}\right)^2\right]^{1/2}. \quad (8)$$

It is easy to see from Eq. (5) that the most "quantum" case (equal probabilities to scatter to the left and to the right) is at $\epsilon = 0$, in fact numerical calculations [11] show that there is an extended state at that energy. Numerical simulations on the network model are performed in the following way: one studies system with fixed width M and periodic boundary conditions in the transverse direction. Multiplying transfer matrices for N slices and then diagonalizing the resulting total transfer matrix , it is possible to extract the smallest Lyapunov exponent λ (the eigenvalues of the transfer matrix are $\exp(\lambda N)$). The localization length ξ_M is proportional to $1/\lambda$. Repeating calculations for different system widths and different energies it is possible to show that the localization length ξ_M satisfies a scaling relation

$$\frac{\xi_M}{M} = f\left(\frac{M}{\xi(\epsilon)}\right). \quad (9)$$

In the QHE the thermodynamic localization length $\xi(\epsilon) \sim |\epsilon|^{-\nu}$ and $\nu = 2.5 \pm 0.5$. This is the main result [11] and it is in a good agreement with experimental data for spin-split resolved levels [13], numerical simulations using other models [14] and semiclassical argument [15, 16] that predicts $\nu = 7/3$.

It is possible to model classical percolation using CC model as well. It was shown [17] that when the relative height of the barriers fluctuate in the infinite range, the percolation becomes classical (no tunneling is allowed) and classical percloation exponent $\nu_{cl} = 4/3$ is retrieved. On the other hand, when the fluctuations are finite, their width acts as irrelevant parameter [18, 19, 20] and does not affect ν.

4 Critical behavior in presence of nuclear polarization

In the present work we modify CC model in the following way. We expect that the presence of a polarized nucleus near a saddle point of the scalar potential will modify a tunneling parameter ϵ in Eq. (3) by changing V_0 to $V_0 \pm V_{hf}$. More, we also expect that due to different relaxation rates (in

the vicinity of impurities of different signs) the following scenario can be realized: nuclei situated near different types of saddle point (nodes of the model) will be polarized in opposite directions, breaking, therefore, isotropy of the system. We model this situation by introducing a parameter $0 \leq p < 1$ describing the probability that there is a polarized nucleus near particular saddle point. Due to the effect of high hyperfine fields descibed above, we, as a rough approximation, can expect that the barrier becomes "infinite", i.e. the transfer matrix at the node is now a unit matrix. On the model language it means that the quasiparticle stays on the same horizontal link (see Fig. 2), and isotropy of the model is therefore broken. Obviously, when $p = 1$ a 2d system is broken into M one-dimensional chains, and, due to the fact that there is no backscattering, all states are extended independent on energy ϵ and system width M. We, therefore, expect the smallest Lyapunov exponent $\lambda = 0$, in contradistinction to the "ordinary" 2d extended state, where λ is finite, and infinite thermodynamic localization length is recovered only after finite size scaling. In this sense $p = 1$ case is close to a 1d metal found for a dirty superconductors with broken time-reversal and spin-rotational symmetries [21].

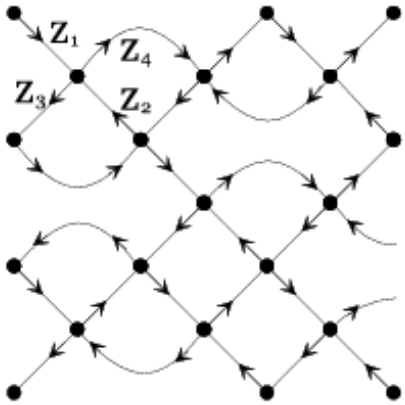

Figure 2: Network model with missing nodes.

Before we present numerical results, let us discuss the possible form for the scaling of the renormalized localization length. Now, when we have "wiped out" on average a fraction p of the nodes, a quasiparticle should travel larger distance (times $1/(1-p)$) in order to experience the same number of scattering events. Therefore, naively, one would expect that the effective system width is now $M(1-p)^{-1}$ and the scaling is

$$\frac{\xi_M}{M} = (1-p)^{-1} f\left(\frac{M}{\xi(\epsilon)}\right). \tag{10}$$

On the other hand, we should take into account that the "missing" node actually does not allow the quasiparticle to propagate in the transverse direction (we have chosen the system in such way that, if there is no scattering, the quasiparticle stays on the same *horizntal* link). Usually for CC model and its generalizations the typical value of the renormalized localization length for the extended state is of the order of 1, meaning that in the extended state the quasiparticle is able to traverse the system of the width M. Therefore, in the present situation we could expect even larger value of ξ_M in the extended state, i.e. $(1-p)^{-\nu}$ dependence with $\nu > 1$.

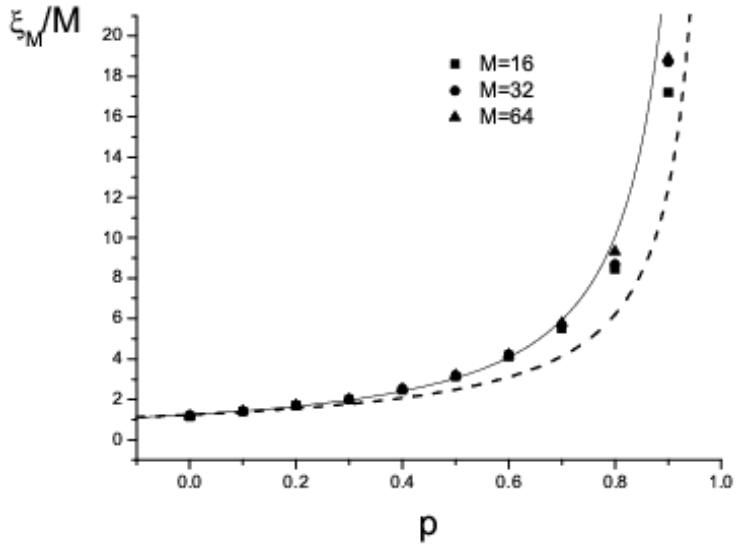

Figure 3: Renormalized localizaton length at critical energy $\epsilon = 0$ as function of the fraction of missing nodes p for different system widths. Solid line is the best fit $1.24(1-p)^{-1.3}$. Dashed line is the fit with "naive" exponent $\nu = 1$.

In order to find both critical exponents we start by studying a p-dependence for $\epsilon = 0$, corresponding to the development of a 2d extended state into a 1d extended state. The results for system widths $M = 16, 32, 64$ are presented on Fig. 3, allowing the following fit

$$\frac{\xi_M}{M} = (1-p)^{-1.3} f(0), \tag{11}$$

where $f(0)$ is the value of the renormalized localization length in the extended state $\epsilon = 0$ for the standard CC model ($p = 0$). This value for the critical exponent is suspiciously close to the classical percolation exponent $\nu_{cl} = 4/3$. We also show visibly worse fit of the data with the "naive" critical exponent $\nu = 1$.

We next use the value ν found in Fig. 2 and study numerically renormalized localization length for various $\epsilon \neq 0$ and $p < 1$. All our data collapse on one curve with abscissa in the form $M/\xi(\epsilon)$ where thermodynamic localization length diverges as $\xi \sim \epsilon^{-\nu_q}$ with quantum percolation exponent $\nu_q \approx 2.5$. The results of the scaling are presented on Fig. 4.

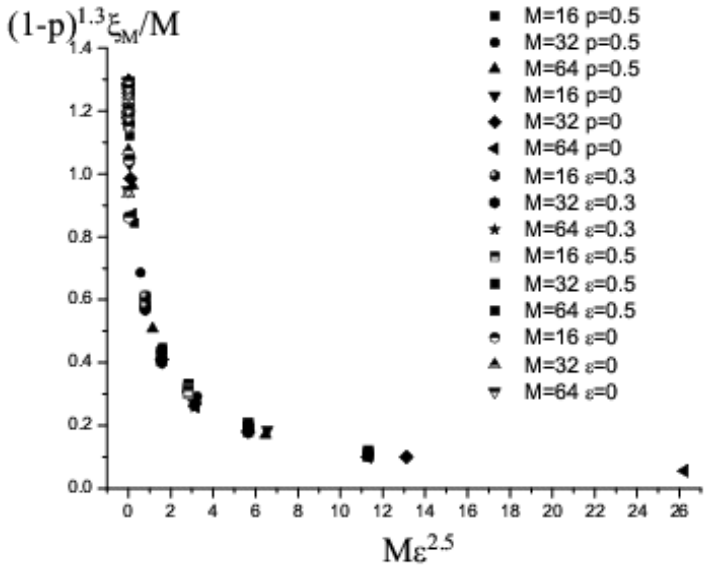

Figure 4: Data collapse for all energies ϵ, system widths M and all fractions $p \neq 1$ of missing nodes.

We argue that one can understand the appearance of the classical percolation exponent in Eq. (7) by considering a quasiparticle on the standard CC model deeply into the localized regime. In this case localization length ξ_M is M-independent, meaning that a quasiparticle does not "feel" the bound-

aries of the system, and its thermodynamic localization length $\xi = \xi_M$. Therefore, a quasiparticle travels on the perimeter of the classical cluster of the typical size ξ. Then by increasing the fraction p of the missing nodes, we increase the size of the classical cluster, actually making it infinite as p approaches 1. Therefore, $(1-p)$ acts as energy in the classical percolation problem, explaining the value $1.3 \approx 4/3$.

Another option to explain the appearance of the value $\nu \approx 1.3$ is the effect of directed percolation. By making a *horizontal* direction preferential, we have introduced an anisotropy into the system. Our result practically coincides with the value of critical exponent for the divergent temporal correlation length in 2d critical nonequilibrium systems, described by directed percolation models [22]. It probably should not come as a suprise if we recollect that each link in the network model can be associated with a unit of time [23].

Finally, all numerical data we have obtained supports the following scaling relation

$$\frac{\xi_M}{M} = (1-p)^{-\nu_{cl}} f(M\epsilon^{\nu_q}), \tag{12}$$

We stress that this is the first result for the network models to produce both quantum and classical percolation exponents form the same data. To summarize, we have studied the influence of nuclear spins on the localization-delocalization transition in quantum Hall systems. We have found that the fraction p of polarized nuclei acts as a relevant parameter, leading to a new scaling relation for the localization length (Eq. (12)).

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When the work on this paper was in its final stages our coauthor Professor Israel Vagner had died after a lengthy and courageous battle with cancer. We wish to express here our admiration of Isya (the name he was called by friends and colleagues) for his devotion to physics, inspiration he had and knew how to ignite in his coworkers, his kindness, his humor and his incredible optimism. This research was just the first step in a project studying the interplay between nuclear magnetism and quantum Hall effect, proposed by Isya. We hope to be able to continue this study, developing Isya's ideas.

References

[1] K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980); D.C. Tsui, H.L. Störmer, and A.C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982); R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[2] R. Prange and S. Girvin, *The Quantum Hall Effect*, (Berlin, Shpringer,1983); *Perspectives in Quantum Hall Effect* Eds: S. Das Sarma and A.P. Pinczuk (Wiley, New York, 1997).

[3] S.V. Iordansky, Solid State Commun., **43**, 1(1982); S. Luryi and R. Kazarinov, Phys.Rev. B**27**, 1386 (1983); R. Joint, J. Phys. C.:Solid State Physics, **18**, L331(1985);I.D. Vagner and M. Pepper, Phys. Rev. B**37**, 7147 (1988).

[4] I.D. Vagner and T. Maniv, Phys. Rev., Lett. **61**, 1400 (1988); I.D. Vagner, T. Maniv, *Hyperfine interactions in Quantum Hall systems*, Review, Physica B**204**, 141(1995); See for a recent review: I.D. Vagner, *Nuclear Spintronics: Quantum Hall and Nanosystems*, HAIT Journal of Science and Engineering **1**, 152 (2004).

[5] A. Berg, M. Dobers, R.R. Gerhardts, and K.v. Klitzing, Phys. Rev. Lett. **64**, 2563 (1990); J.N. Smet, R.A. Deutschmann, F. Ertl, W. Wegscheider, G. Abstreiter, and K. von Klitzing, Nature **415**, 281 (2002).

[6] Go Yusa, Koji Muraki, Key Takashima, Katsushi Nashimoto,Yoshiro Kirayama, Nature, **430**, 439 (2004); G. Yusa, K. Hashimoto, K. Muraki,T. Saku,and Y. Hirayama, Phys. Rev. B**69,** 161302 (2004); M.Xiao, I. Martin,Eli Yablonovichi, H.W. Jiang, Nature **430**,435 (2004).

[7] A. Abragam, *The Principles of Nuclear Magnetism* (Clarendon, Oxford, 1961); C.P. Slichter, *Principles of magnetic resonance* (Springer Verlag, Berlin, 3rd ed, 1991).

[8] Yu. V. Pershin, S.N. Shevchenko, I.D. Vagner, and P. Wyder, Phys. Rev. B **66**, 035303 (2002).

[9] S.V. Iordanskii, S.V. Meshkov, and I.D. Vagner, Phys. Rev. B **44**, 6554 (1991).

[10] Yu.A. Bychkov, S.V. Iordanskii, and G.M. Eliashberg, Pis'ma Zh.Eksp.Teor.Fiz.**33**, 152 (1981); Sov. Phys-JETP Lett. **33**, 143 (1981).

[11] J.T. Chalker and P.D. Coddington, J. Phys. C **21**, 2665 (1988).

[12] H.A. Fertig and B.I. Halperin, Phys. Rev. B **36**, 7969 (1987).

[13] S. Koch, R.J. Haug, K. v. Klitzing and K. Ploog, Phys. Rev. Lett. **67**, 883 (1991).

[14] B. Huckestein and B. Kramer, Phys. Rev. Lett **64**, 1437 1990.

[15] G.V. Milnikov and I.M. Sokolov, JETP Lett. **48**, 536 (1988).

[16] V. Kagalovsky, B. Horovitz, and Y. Avishai, Europhys. Lett. **31**, 425 (1995).

[17] D.H. Lee, Z. Wang, and S. Kivelson, Phys. Rev. Lett. **70**, 4130 (1993).

[18] B. Huckestein, Phys.Rev.Lett. **72**, 1080 (1994).

[19] J.T. Chalker and J.F.G. Eastmond (unpublished); J.F.G. Eastmond, Ph. D. thesis, Oxford University, 1992.

[20] V. Kagalovsky, Ph. D. thesis, Ben Gurion University, 1995.

[21] J.T. Chalker, N. Read, V. Kagalovsky, B. Horovitz, Y. Avishai, and A.W.W. Ludwig, Phys. Rev. B **65**, 012506 (2002).

[22] H. Hinrichsen, Adv. Phys. **49**, 815 (2000); G. Odor, Rev. Mod. Phys. **76**, 663 (2004); S. Lübeck, Int. J. Mod. Phys. B **18**, 3977 (2004).

[23] C.M. Ho and J.T. Chalker, Phys. Rev. B **54**, 8708 (1996).